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**STRAIN GAGE DATA PROCESSING FOR ESTIMATING
STRESS DISTRIBUTIONS AROUND PIPE CROSS
SECTIONS**

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STRAIN GAGE DATA PROCESSING FOR ESTIMATING
STRESS DISTRIBUTIONS AROUND PIPE CROSS SECTIONS

by

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ABSTRACT

This thesis presents a method for estimating the distribution of stress components around the outer circumference of a cross section of a pipe, from the readings of strain gage elements arbitrarily positioned and oriented around this circumference. A least-squares procedure is used to obtain best estimates of the coefficients of Fourier expansions describing such distributions. A digital computer program was developed for applying the method. Data for testing the method and program were generated by a computer program using the best available theory of stress analysis in pipes. Methods for adding random errors to the data were adapted and used for closer simulation of actual situations.

A second problem treated in this thesis is that of inferring the loading acting through the cross section of a straight pipe of concentric bore from known stresses at points on the external surface of the cross section which is presumed to be distant from stress concentrations. It is shown that this inference can be made from stresses at only three points of the cross section. A digital computer program was developed to do this.

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1. Introduction.

In a thesis presented to the Naval Postgraduate School in May, 1966, LT Joseph W. Koch, Jr., USN, did pioneer work on the problem of obtaining optimal inferences of stresses in submarine main sea water piping, from strain gage readings taken during dockside hydrostatic tests and during submerged operations. This work provided a sequential procedure for determining such stresses, but it also indicated the need for many related investigations.

Two of these are treated in the present thesis. The first is that of inferring, in an optimal way, the distributions of stresses around the outer circumference of a given cross section of pipe from observations of strain obtained from an arbitrary distribution of strain gage elements around that circumference. There are several reasons for being interested in this problem. One reason is that even though the gage elements may be arranged in special patterns intended to simplify determining stresses (for example three-gage rosette arrangements), failure of any of the elements might make it difficult to make effective use of valid information obtained from closely related gages. Another reason is the ability to infer stresses from arbitrarily arranged gages, which is a logical step in investigating the most effective and most economical arrangements of gage elements.

Briefly, this problem is solved herein by constructing Fourier series representing the desired stress components. Theoretical considerations are used to evaluate the coefficients of the series in an optimal way, minimizing the sum of the squares of the residuals. A digital computer program was developed to support the theory.

The second problem treated here is that of inferring the force and moment components applied through a cross section of pipe and the internal pressure acting inside the pipe. It is shown that these values may be inferred from a knowledge of the state of stress at only three arbitrarily chosen points at the external circumference of the cross section, and a digital computer program for making the corresponding evaluations was developed.

The solution of this problem assumes that the stress distribution is that given by what may be regarded as the best available current theory for the state of stress in a section of uniform pipe having concentric bore and remote from stress concentrations and other disturbances in the stress pattern.

A digital computer program to generate numerical data from the best available theory was developed. This data was used to test and verify the theory and the computer programs. Suggestions of how to extend the present work are given in Section 4.

The author would like to express his sincere appreciation to Dr. John E. Brock, Professor of Mechanical Engineering, Naval Postgraduate School, for his continued patience, efforts and most capable guidance while acting as faculty advisor in this work.

2. Least-Squares Estimation of Stress Distributions at the External Circumference of a Pipe by Means of Fourier Expansions, Using Strain Gage Data.

The usual method of estimating the stress distribution at the surface of a body is by taking readings of strain gages grouped in rosettes, to determine local principal stresses. Unless the strain rosettes have a redundancy of gage elements, when one or more of them fail, it is difficult to make effective use of the information provided by the elements which do not fail.

The work that follows presents a method to estimate the stress distribution around an external circumference of a cross section of pipe, along a sequence of strain gage elements, which may or may not be grouped in rosettes. If one or more gage elements fail, the information from the others is still useful.

It is assumed that strain gage elements are arbitrarily placed

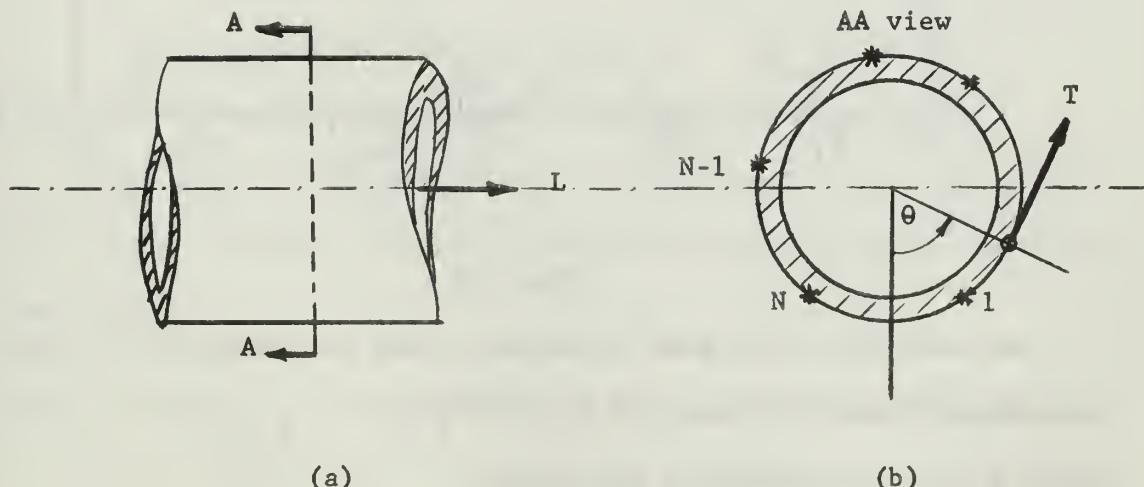


Fig. 2.1.

and arbitrarily oriented around the external circumference of a cross section of pipe so as to measure the strain produced by application of

loading. Assume that there are N such elements as indicated in Fig.

2.1.

Define two orthogonal directions at the cross section, one normal to it, called longitudinal and represented by the letter L, and the other tangential to the external circumference, called tangential and represented by T; Fig. 2.1.b. Also define the angle θ as the angle between the vertical line that passes downward through the center of the cross section, and the radial line that passes through any specified point of the external circumference, measured as indicated in Fig. 2.1.b.

Measure the angular orientation of each gage element in the counter-clockwise direction from the tangential axis T, and call this angle ϕ .

A set of parameter-pairs can then be formed

$$(\theta_i, \phi_i) \quad i = 1, 2, \dots, N$$

which describe the position and orientation of each element, Fig. 2.2.

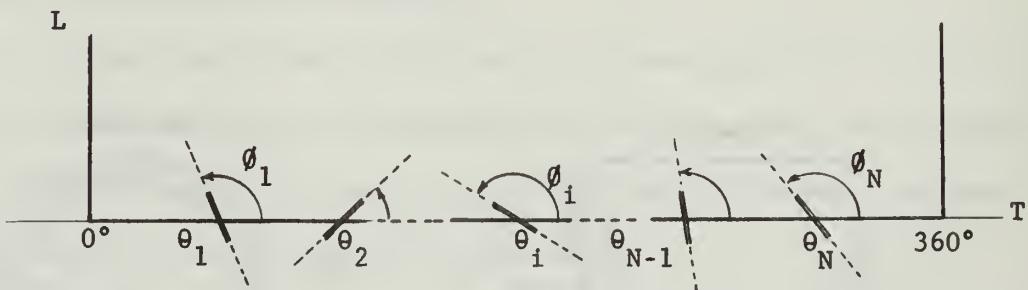


Fig. 2.2.

At each one of the gage locations, there are tangential, longitudinal, and shear strains, $(\epsilon_T)_i$, $(\epsilon_L)_i$, and $(\epsilon_{TL})_i$ respectively. The strains ϵ_y and ϵ_z along axis y and z, rotated the angle ϕ from the tangential direction

Fig. 2.3., are related to the local

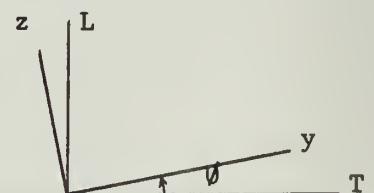


Fig. 2.3.

strains ϵ_T , ϵ_L and ϵ_{TL} by the strain tensor in the following way

$$\begin{bmatrix} \epsilon_y & \frac{1}{2}\epsilon_{yz} \\ \frac{1}{2}\epsilon_{yz} & \epsilon_z \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \epsilon_T & \frac{1}{2}\epsilon_{TL} \\ \frac{1}{2}\epsilon_{TL} & \epsilon_L \end{bmatrix} \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

The corresponding relations for the strains $(\epsilon_y)_i$ and $(\epsilon_z)_i$ along and transverse to the i th strain gage axis, are then

$$(\epsilon_y)_i = (\epsilon_T)_i \sin^2 \theta_i + (\epsilon_L)_i \cos^2 \theta_i + (\epsilon_{TL})_i \sin \theta_i \cos \theta_i \quad (2.1.1)$$

$$(\epsilon_z)_i = (\epsilon_T)_i \cos^2 \theta_i + (\epsilon_L)_i \sin^2 \theta_i - (\epsilon_{TL})_i \sin \theta_i \cos \theta_i \quad (2.1.2)$$

At the external surface, strains are related to the corresponding stresses as follows

$$\epsilon_T = (s_T - v s_L) / E \quad (2.2.1)$$

$$\epsilon_L = (s_L - v s_T) / E \quad (2.2.2)$$

$$\epsilon_{TL} = s_{TL} / G \quad (2.2.3)$$

where s is stress

v is Poisson's ratio

E is Young's modulus of elasticity

G is the shear modulus of elasticity

Substituting these expressions for the strains in the relations (2.1) for the strains along and transverse to the i th gage element and simplifying

$$(\epsilon_y)_i = (s_T)_i [1 - (v+1)\sin^2 \theta_i] / E + (s_L)_i [1 - (v+1)\cos^2 \theta_i] / E + (s_{TL})_i \sin 2 \theta_i / (2G) \quad (2.3.1)$$

$$(\epsilon_z)_i = (s_T)[1 - (v + 1)\cos^2\phi_i]/E + (s_L)_i[1 - (v + 1)\sin^2\phi_i]/E - (s_{TL})_i \sin^2\phi_i/(2G) \quad (2.3.2)$$

A strain gage element is sensitive not only to the strain along its own axis but also, to a small extent, to the strain at right angles to this axis. The transverse sensitivity is usually hard to obtain. It is rarely available from the manufacturer and may have to be inferred from an experiment. Nevertheless, it may be of importance and it is included in the theoretical development that follows.

For a particular gage element properly mounted on a given definite material, the relative change in electrical resistance, which is sensed by the strain indicator (or recorder), is given by a formula of the sort

$$\frac{\Delta R}{R} = F_y \epsilon_y + F_z \epsilon_z \quad (2.4)$$

where R is the electrical resistance of the element, ϵ_y and ϵ_z are the strains along and perpendicular to its axis, and F_y and F_z are coefficients which depend on the characteristics of the gage and the properties of the material upon which it is mounted.

The ratio

$$k = F_z / F_y \quad (2.5)$$

is called the transverse sensitivity (factor) of the gage, and the quantity

$$F' = (1 - k^2) F_y$$

is called the gage factor (which should be set on the sensor instrument). F' is the ratio of unit change in electric resistance to the quantity e , (indicated strain) sensed by the instrumentation.

Thus

$$\begin{aligned} e &= \left(\frac{\Delta R}{R} \right) / F' \\ &= (F_y \epsilon_y + F_z \epsilon_z) / [(1 - k^2) F_y] \\ &= (\epsilon_y + k \epsilon_z) / (1 - k^2) \end{aligned} \quad (2.7)$$

This is what the instrumentation should indicate if there were no errors. Clearly for a gage with no transverse sensitivity, $k = 0$, and

$$e = \epsilon_y.$$

After substituting the expressions (2.3) for $(\epsilon_y)_i$ and $(\epsilon_z)_i$ in equation (2.7) and simplifying, the theoretical indicated strain e_i sensed along the i th strain gage element is given by

$$\begin{aligned} e_i &= (s_T)_i [(1 - kv) - (1 - k)(1 + v) \sin^2 \theta_i] / [E(1 - k^2)] + \\ &+ (s_L)_i [(1 - kv) - (1 - k)(1 + v) \cos^2 \theta_i] / [E(1 - k^2)2] + \\ &+ (s_{TL})_i (1 - k) \sin 2\theta_i / [2G(1 - k^2)] \end{aligned} \quad (2.8)$$

Let the coefficients of the stresses in the above expression be represented by

$$A(\theta_i) = A_i = [(1 - kv) - (1 - k)(1 + v) \sin^2 \theta_i] / [E(1 - k^2)] \quad (2.9.1)$$

$$B(\theta_i) = B_i = [(1 - kv) - (1 - k)(1 + v) \cos^2 \theta_i] / [E(1 - k^2)] \quad (2.9.2)$$

$$C(\theta_i) = C_i = (1 - k) \sin 2\theta_i / [2G(1 - k^2)] \quad (2.9.3)$$

Then equation (2.8) can be put in the form

$$e_i = A_i (s_T)_i + B_i (s_L)_i + C_i (s_{TL})_i \quad (2.10)$$

Any one of the stresses $(s_T)_i$, $(s_L)_i$ and $(s_{TL})_i$ has continuous and finite values around the external circumference of the pipe cross

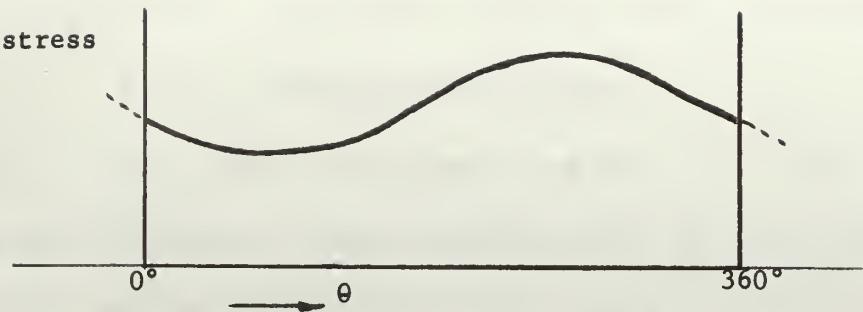


Fig. 2.4.

section, and can be represented as in Fig. 2.4.

The functional relation between stress and the angular position θ can be expanded in an infinite Fourier series¹

$$(s_T)_i = \sum_{n=0}^{\infty} (s_{TA})_n \cos n\theta_i + \sum_{n=0}^{\infty} (s_{TB})_n \sin n\theta_i \quad (2.11.1)$$

$$(s_L)_i = \sum_{n=0}^{\infty} (s_{LA})_n \cos n\theta_i + \sum_{n=0}^{\infty} (s_{LB})_n \sin n\theta_i \quad (2.11.2)$$

$$(s_{TL})_i = \sum_{n=0}^{\infty} (s_{TLA})_n \cos n\theta_i + \sum_{n=0}^{\infty} (s_{TLB})_n \sin n\theta_i \quad (2.11.3)$$

where

$(s_{TA})_n$, $(s_{TB})_n$, $(s_{LA})_n$, $(s_{LB})_n$, $(s_{SHA})_n$, and $(s_{SHE})_n$ are

as yet unknown numerical coefficients.

Sufficiently accurate values can be obtained if the expansion is carried to a finite number of terms, say P terms, in which case the index n would reach a maximum value of $P/2$. Call this value M . Then

$$(s_T)_i \approx \sum_{n=0}^{M} (s_{TA})_n \cos n\theta_i + \sum_{n=0}^{M} (s_{TB})_n \sin n\theta_i \quad (2.12.1)$$

¹ The conventional expansion of a Fourier series is a constant term followed by a summation from 1 to ∞ . The same is obtained by eliminating the constant term and making summations from 0 to ∞ . This latter form will be used in this work to facilitate mathematical manipulations and later programming for a digital computer.

$$(s_L)_i \approx \sum_{n=0}^M (s_{LA})_n \cos n\theta_i + \sum_{n=0}^M (s_{LB})_n \sin n\theta_i \quad (2.12.2)$$

$$(s_{TL})_i \approx \sum_{n=0}^M (s_{TLA})_n \cos n\theta_i + \sum_{n=0}^M (s_{TLB})_n \sin n\theta_i \quad (2.12.3)$$

Note that this expansion contains $6(M + 1)$ Fourier coefficients.

If these expressions for the stresses are substituted into equation (2.10) for the strain e_i , theoretically sensed by the i th strain gage element,

$$e_i \approx A_i [\sum_{n=0}^M (s_{TA})_n \cos n\theta_i + \sum_{n=0}^M \dots] + \\ + B_i [\sum_{n=0}^M \dots + \sum_{n=0}^M \dots] + \quad (2.13) \\ + C_i [\sum_{n=0}^M \dots + \sum_{n=0}^M (s_{TLB})_n \sin n\theta_i]$$

Call e_i^* the strain actually measured by the i th strain gage element, and E_i the difference between that measured strain and e_i , the strain that theoretically it should have measured, so that this error E_i is expressed as

$$E_i = e_i - e_i^* \quad (2.14)$$

Let R be the sum of the squares of the errors, that is

$$R = \sum_{i=1}^N (E_i)^2 \\ = \sum_{i=1}^N (e_i - e_i^*)^2 \\ = \sum_{i=1}^N (e_i)^2 - 2 \sum_{i=1}^N e_i e_i^* + \sum_{i=1}^N (e_i^*)^2 \quad (2.15)$$

To minimize R , partial derivatives respect to each of the $6(M + 1)$ Fourier coefficients are taken and set equal to zero:

$$\frac{\partial R}{\partial D_k} = 0$$

where $D_k = (s_{TA})_k, (s_{TB})_k, (s_{LA})_k, (s_{LB})_k, (s_{TLA})_k, (s_{TLB})_k$ in turn

for $k = 0, 1, 2, \dots, M$

Taking derivatives of R from expression (2.15)

$$\frac{\partial R}{\partial D_k} = 0 = 2 \sum_{i=1}^N e_i \frac{\partial e_i}{\partial D_k} - 2 \sum_{i=1}^N e_i^* \frac{\partial e_i}{\partial D_k} \quad (2.16)$$

Simplifying this

$$\sum_{i=1}^N e_i \frac{\partial e_i}{\partial D_k} = \sum_{i=1}^N e_i^* \frac{\partial e_i}{\partial E_k} \quad (2.17)$$

Taking derivatives of e_i from expression (2.13)

$$\frac{\partial e_i}{\partial (s_{TA})_k} = A_i \cos k\theta_i \quad \text{for } k = 0, 1, 2, \dots, M \quad (2.18.1)$$

$$\frac{\partial e_i}{\partial (s_{TB})_k} = A_i \sin k\theta_i \quad (2.18.2)$$

$$\frac{\partial e_i}{\partial (s_{LA})_k} = B_i \cos k\theta_i \quad (2.18.3)$$

$$\frac{\partial e_i}{\partial (s_{LB})_k} = B_i \sin k\theta_i \quad (2.18.4)$$

$$\frac{\partial e_i}{\partial (s_{TLA})_k} = C_i \cos k\theta_i \quad (2.18.5)$$

$$\frac{\partial e_i}{\partial (s_{TLB})_k} = C_i \sin k\theta_i \quad (2.18.6)$$

Note that there are $6(M + 1)$ derivatives of this form. Substituting in equation (2.17) the expression for e_i from equation (2.13)

$$\begin{aligned} \sum_{i=1}^N \left\{ A_i \left[\sum_{n=0}^M (s_{TLB})_n \sin n\theta_i + \dots \right] + \dots + C_i \left[\dots \right. \right. \\ \left. \left. \dots + \sum_{n=0}^M (s_{TLB})_n \sin n\theta_i \right] \right\} \frac{\partial e_i}{\partial D_k} = \sum_{i=1}^N e_i^* \frac{\partial e_i}{\partial D_k} \end{aligned} \quad (2.19)$$

which is the same as

$$\sum_{n=0}^M (s_{TA})_n \left[\sum_{i=1}^N A_i \cos n\theta_i \frac{\partial e_i}{\partial D_k} \right] + \sum_{n=0}^N (s_{TB})_n \left[\sum_{i=1}^N A_i \sin n\theta_i \frac{\partial e_i}{\partial D_k} \right] + \\ + \dots + \sum_{n=0}^M (s_{TLB})_n \left[\sum_{i=1}^N C_i \sin n\theta_i \frac{\partial e_i}{\partial D_k} \right] = \sum_{i=1}^N e_i^* \frac{\partial e_i}{\partial D_k} \quad (2.20)$$

for $k = 0, 1, 2, \dots, M$

When the index n in the Fourier expansions is equal to zero, the terms containing $\sin(n\theta_i)$ are zero. This makes the coefficients $(s_{TB})_0$, $(s_{LB})_0$ and $(s_{TLB})_0$ also zero, reducing the Fourier coefficients to $(6M + 3)$ and the same number of derivatives of the form $\frac{\partial e_i}{\partial D_k}$. There will be $(6M + 3)$ equations in each of which appear all the Fourier coefficients. This makes a set of $(6M + 3)$ equations with the same number of unknowns, (the Fourier coefficients), which can be solved.

Once the Fourier coefficients are solved for, they can be substituted in expressions (2.7) for the stresses. The stress distributions s_T , s_L and s_{TL} are defined then as functions of the angle θ over the complete external circumference of the cross section.

A digital computer program was developed to apply the theory of the preceding text. It was tested using a mathematical model of a cantilevered pipe. The program is called TERESITA and is presented in Appendix B of this thesis.

It was assumed that strain gages having a zero transverse sensitivity factor were placed at the external circumference of cross section No. 1, see Fig. A.1. in Appendix A. Four arrangements of gage elements

and several numbers of Fourier coefficients were used.

The first arrangement consisted of twelve strain gage rosettes equally spaced 30° from each other around the cross section. Each rosette has three gage elements oriented at 0° , 45° and 90° from the positive tangential axis. Runs were made with 11, 9, 7, 5 and 3 Fourier coefficients for each stress component.

The second arrangement consisted of twelve rosettes spaced as above but with elements oriented at 0° , 60° , and 120° . Runs were made for the same number of Fourier coefficients.

The third arrangement consisted of eight strain rosettes equally spaced 45° from each other. The rosettes had four gage elements oriented at 0° , 45° , 90° and 135° from the positive tangential axis. Runs were made for the same number of Fourier coefficients.

As input data for testing the program, stress distributions produced by the loading were evaluated at the external circumference of cross section No. 1. From the stresses, the theoretical strain that the gage element would sense, was computed. The stress distributions recovered by the program agreed in all cases with the input information. Results using eleven Fourier coefficients (for each stress component s_T , s_L and s_{TL}) are almost identical with those using only three coefficients.

Additional runs were made using the second arrangement of gages, but eliminating arbitrarily the readings of some of them. This is what would have to be done in an actual case if some gage elements were known to have failed. Results are satisfactorily close to the preceding ones. Other runs were made using a strain gage transverse sensitivity factor of 1%. The program recovered likewise the input information.

The fourth gage arrangement had one gage element every 10° around

the circumference with different orientation for each gage. Runs were made using the same number of coefficients.

All the runs described above were repeated introducing in the gage element readings small random errors so to simulate actual situations. Calculated loadings differed from the input loadings to a degree which depended in each case on the amount of error introduced.

The program and some sample numerical results of the runs made are presented in Appendix B of this thesis.

A more detailed discussion of the effects of changing the number of Fourier coefficients and gage element arrangement is presented in Appendix E.

3. Inference of the Loading Acting at a Cross Section of a Pipe of Concentric Bore from Stresses at the External Circumference of that Cross Section.

If stresses are known at a sufficient number of points around the external circumference of the cross section of a pipe of concentric bore at a distance from stress risers and concentrated loads, the loading acting at the cross section can be determined from theory.

It is intended to analyse the effects that such loads produce at a general point P under the conditions specified above. From this analysis, general conclusions will be reached which lead to a mathematical procedure to determine the loads.

Consider a cross section of pipe as specified above, Fig. 3.1. Choose reference axes x, y and z with the center point O as origin. Let

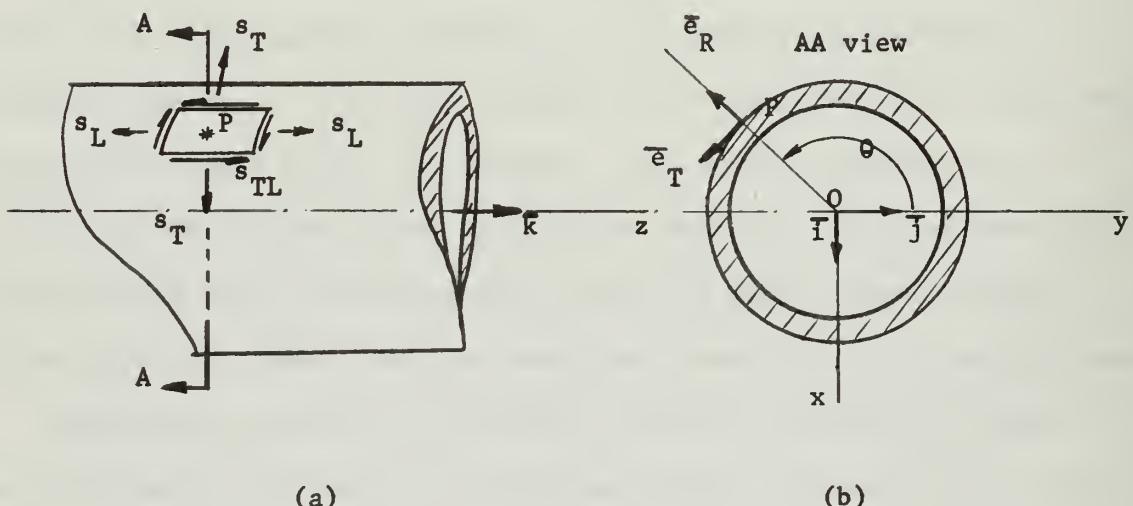


Fig. 3.1.

\bar{i} , \bar{j} and \bar{k} be unit vectors along the positive x, y and z axis so to form a right handed triad. Let \bar{e}_R be a unit vector along the outwards direction of the radius that passes through the point P, and \bar{e}_T another unit vector so that \bar{e}_R , \bar{e}_T and \bar{k} form a right handed triad. The angle θ is

measured from \bar{i} counterclockwise to \bar{e}_R . Consider the differential area surrounding the point P. Assume that tensile stresses s_T and s_L (along the directions of \bar{e}_T and \bar{k} respectively) and shear stress s_{TL} , are produced by internal pressure p and by a general force \bar{F} and a general moment \bar{M} acting on the cross section at the center point O.

Let the components of \bar{F} and \bar{M} in the \bar{e}_R , \bar{e}_T and \bar{k} triad be (F_R, F_T, F_z) and (M_R, M_T, M_z) respectively, and in the \bar{i} , \bar{j} , \bar{k} triad, (F_x, F_y, F_z) and (M_x, M_y, M_z) .

From strength of materials considerations reviewed in reference 5, the stresses produced by p , \bar{F} and \bar{M} at the external point P are:

$$s_T = 2 p a^2 / (b^2 - a^2) \quad (3.1)$$

$$s_L = -M_T b / I + F_z / A \quad (3.2)$$

$$s_{TL} = M_z b / (2 I) + D F_T \quad (3.3)$$

where

a = internal radius of the cross section

b = external radius of the cross section

A = cross sectional area

I = moment of inertia of the cross sectional area about a diameter

$$D = [2 n a^2 + (2n - 2)b^2] / [4I(1 + v)]$$

v = Poisson's ratio

$$n = v + 3/2$$

Making use of the two dimensional coordinate transformation tensor

$$\begin{bmatrix} M_R \\ M_T \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} M_x \\ M_y \end{bmatrix}$$

M_T can be expressed as

$$M_T = -M_x \sin \theta + M_y \cos \theta \quad (3.4)$$

Similarly,

$$F_T = -F_x \sin \theta + F_y \cos \theta \quad (3.5)$$

The expression (3.2) for s_L can be transformed to

$$s_L = (1/A)F_z + (b/I)\sin \theta M_x - (b/I)\cos \theta M_y \quad (3.6)$$

Multiplying through by (I/b)

$$(I/b)s_L = (I/(Ab))F_z + (\sin \theta)M_x - (\cos \theta)M_y \quad (3.7)$$

Letting $(I/b) = g$, and $(I/(Ab)) = h$, both of which are cross sectional constants, substituting above and changing sides

$$h F_z + \sin \theta M_x - \cos \theta M_y = g s_L \quad (3.8)$$

The preceding theory is applicable to a point on the exterior surface of a piece of straight, uniform, concentric pipe at a sufficiently great distance from its ends or other connections which might introduce local disturbances into the stress distribution. Presuming that these conditions are satisfied, we next turn to the matter of trying to infer the loading conditions at the cross section, i.e., F_x , F_y , F_z , M_x , M_y , M_z and p , from a knowledge of the stresses at several points on the external surface of the cross section.

For a general point P , s_L and θ are known and the only unknowns in equation (3.8) are F_z , M_x and M_y . Taking three such points at the cross section, the three unknowns can be determined proceeding as follows.

Assume that the angular positions for the points are θ_1 , θ_2 and θ_3 and the axial stresses are s_1 , s_2 and s_3 respectively. A set of equations can then be formed

$$h F_z + s\theta_1 M_x - c\theta_1 M_y = gs_1 \quad (3.9.1)$$

$$h F_z + s\theta_2 M_x - c\theta_2 M_y = gs_2 \quad (3.9.2)$$

$$h F_z + s\theta_3 M_x - c\theta_3 M_y = gs_3 \quad (3.9.3)$$

where the letters s and c with no subscripts are abbreviations for sin and cos respectively. These abbreviations will be used throughout this section.

The set of equations (3.9) is solved applying Cramer's rule

$$\text{Denominator} = \begin{vmatrix} h & s\theta_1 & -c\theta_1 \\ h & s\theta_2 & -c\theta_2 \\ h & s\theta_3 & -c\theta_3 \end{vmatrix} = -h(s(\theta_1 - \theta_2) + s(\theta_2 - \theta_3) + s(\theta_3 - \theta_1)) \quad (3.10)$$

$$\text{Numerator of } F_z = \begin{vmatrix} gs_1 & s\theta_1 & -c\theta_1 \\ gs_2 & s\theta_2 & -c\theta_2 \\ gs_3 & s\theta_3 & -c\theta_3 \end{vmatrix} = -g(s_1s(\theta_2 - \theta_3) + s_2s(\theta_3 - \theta_1) + s_3s(\theta_1 - \theta_2)) \quad (3.11)$$

$$\text{Numerator of } M_x = \begin{vmatrix} h & gs_1 & -c\theta_1 \\ h & gs_2 & -c\theta_2 \\ h & gs_3 & -c\theta_3 \end{vmatrix} = -hg(c\theta_1(s_3 - s_2) + c\theta_2(s_1 - s_3) + c\theta_3(s_2 - s_1)) \quad (3.12)$$

$$\text{Numerator of } M_y = \begin{vmatrix} h & s\theta_1 & gs_1 \\ h & s\theta_2 & gs_2 \\ h & s\theta_3 & gs_3 \end{vmatrix} = -hg(s\theta_1(s_3 - s_2) + s\theta_2(s_1 - s_3) + s\theta_3(s_2 - s_1)) \quad (3.13)$$

The values of F_z , M_x and M_y are found by dividing the respective numerator by the denominator.

The remaining components of the vectors \bar{F} and \bar{M} appear in the expression (3.3) for s_{TL} , which by using equation (3.5) can be transformed to

$$s_{TL} = (b/2I)M_z - D\sin \theta F_x + D\cos \theta F_y \quad (3.14)$$

Multiplying through by $(1/D)$, letting $(-1/D) = g^*$, $(-b/2ID) = h^*$, both cross sectional constants, substituting in (3.14) and changing sides

$$h^* M_z + \sin \theta F_x - \cos \theta F_y = g^* s_{TL} \quad (3.15)$$

This expression is of the same form as (3.8) which led to the evaluation of F_z , M_x and M_y by using information from three points. In this case the unknowns are M_z , F_x and F_y which can be evaluated by using the same three points and applying the same method. In this form the loads \bar{F} and \bar{M} are completely determined.

Since s_T is produced by the internal pressure p alone, it can be solved for from equation (3.1) with the information from any one of the three points used to evaluate \bar{F} and \bar{M}

$$p = s_T(b^2 - a^2)/(2a^2) \quad (3.16)$$

From the above development, it can be concluded that theoretically the loads \bar{F} , \bar{M} and p acting at the cross section can be determined from known stresses at three points of the external surface of the cross section.

A digital computer program called TIBISAY was developed to apply the preceding treatment. The program was tested using a mathematical model of a cantilevered pipe as in Section 2 of this thesis. Stresses generated at cross section No. 1, at points 0° , 120° and 240° from the

vertical downwards direction were used. The recovered loads were almost identical with the input ones producing the stresses. Deviations are due to small round-off errors introduced by the computer. The program and results of the test are presented in Appendix C of this thesis.

4. Conclusions.

The results obtained in the tests for applicability of the theories developed in this thesis indicate that the proposed methods can be used for analysing stresses in pipes.

There are, however, many factors influencing the validity of the results. Some of them are related to the amount of error introduced when determining geometries and material properties. Others are due to proximity to concentrated loads and stress concentrations. Other factors are directly related to the degree of redundancy of strain gage data, the number of gages used and their arrangement around the cross section, and the number of Fourier coefficients to be used. Also, it may be that the least-squares method used does not produce the best results; perhaps, by minimizing some other function of error, better results could be obtained.

First steps were taken to investigate further some of these factors. Appendix E of this thesis presents results of a survey made concerning the influence of the number of Fourier coefficients used in the accuracy of results. There are also preliminary results of the influence of the number of gage elements and their arrangement around the cross section. Due to time limitations, further investigations were not conducted.

The digital computer programs included in this thesis can be used to investigate deeper into the field. Some refinements of the methods can be added. It is believed that the following points should not be neglected.:

- a. Other criteria for minimizing errors can be investigated. A discussion of such criteria is given in reference 7 of this thesis. It is likely that some of the methods described there may have application in the pipe stress problem considered herein.

b. One may investigate the question of the minimum number of strain gage elements necessary to obtain acceptable stress distributions, and their orientation around the cross section. Similarly, the optimum number of elements and preferred orientation could be investigated.

c. While observing the results for a determined gage arrangement, it was noticed that good results were obtained using from 3 to 7 Fourier coefficients, but more than that produced bad results; using again 5 coefficients and changing the gage arrangement, good results were obtained for some arrangements and bad results for others. Some of these results are presented in Appendix G of this thesis. Further study could be made of the optimal number of Fourier coefficients to be employed.

d. A method for data rejection should be incorporated in the theory and computer programs. There are instances when it is obvious that a strain gage element has failed and the reading of that gage should be neglected. Usually bad gages are not easily identified and bad data is processed along with good data. There is the possibility of incorporating a data rejection procedure in the method developed in Section 2 of this thesis. One suggestion for such a procedure is given in the following.

The estimated Fourier coefficients of the stress distributions are evaluated from a minimization of the sum of the squares of the errors, i.e., the differences between the gage element readings and what they should have read theoretically. We can assume that these errors are random having a statistical normal distribution with mean and standard deviation which could be computed. Establishing a level of significance for a deviation to be sufficiently apart from the mean, one can reject the data having greater deviations and recompute the stress distributions with the remaining data. Similar screenings can be done with the

newly computed stress distributions until the remaining data have deviations under the level of rejection. This theory could be readily used if the redundancy of data were large. Usually this is not the case and one must avoid rejecting data which are not perfect but which are useful and necessary. If the data is known to have a normal curve with a very sharp peak, bad data is easily determined and low significance levels can be used. If the bell is fairly flat, the majority of the deviations have pronounced deviations from the mean and the level of rejection becomes difficult to establish. The writer is inclined to think that there are criteria that may satisfy all conditions. Once they are found, the theory could be applied to this problem and to similar ones.

5. Bibliography.
1. Koch, J. W., Jr., Analysis of Strain Gage Data from Submarine MSW Piping, Thesis, Naval Postgraduate School, May 1966.
2. Timoshenko, S. and Goodier, J. N., Theory of Elasticity, McGraw-Hill Book Company, Inc., 1951.
3. Perry, C. C. and Lissner, H. R., The Strain Gage Primer, McGraw-Hill Book Company, Inc., 1962.
4. Lee, G. H., An Introduction to Experimental Stress Analysis, John Wiley and Sons, Inc., 1950.
5. Blair, N. and Brock, J. E., Sixth Progress Report, Contract N161-26327 with U. S. Naval Academy, 1967.
6. Brock, J. E., Fourth Progress Report, Contract N161-26327 with U. S. Naval Academy, 1966.
7. Villaran Tapia, C. F., Determination of Optimal Compliance and Stiffness Matrices from Experimental Data, Thesis, Naval Post-graduate School, September 1967.

APPENDIX A

DATA GENERATION THEORY AND RELATED DIGITAL COMPUTER PROGRAMS

The data used to test the applicability of the theories developed in this thesis was generated for a mathematical model of a cantilevered pipe of concentric bore subjected to internal pressure p and general force \bar{F} and general moment \bar{M} applied to the tip. Details of this pipe are shown in Fig. A.1. The following material properties, pipe dimensions, and loads were specified:

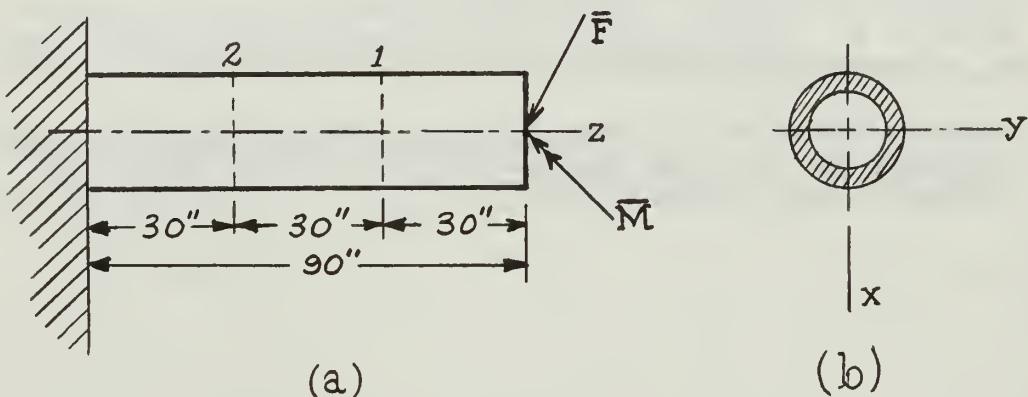


Fig. A.1.

$$E = 30,000 \text{ ksi}$$

$$\nu = 0.3$$

$$\text{Specific weight} = 0.283 \text{ lb/in}^3$$

$$\text{External diameter} = 12.0 \text{ in}$$

$$\text{Internal diameter} = 10.0 \text{ in}$$

$$F_x = F_y = F_z = 100.0 \text{ lb}$$

$$M_x = M_y = M_z = 100.0 \text{ lb-in}$$

$$p = 100.0 \text{ psi}$$

Subroutine CANTIL presented herein in this Appendix, was used to evaluate the tangential, longitudinal and shear stress distributions

at the external circumference of cross sections 1 and 2 of this pipe.

The stress distributions were used in turn to supply the data required for the tests specified in other parts of this thesis.

This subroutine is used by either of the (main) programs TERESITA or TIBISAY described in Appendices B and C. The arrangement of the card input is indicated in Fig. B.1. of this thesis. Description and specifications for CANTIL follow this page.

DESCRIPTION OF SUBROUTINE CANTIL

Subroutine CANTIL evaluates the state of stress at any point of a cantilevered straight pipe of concentric bore, with internal pressure and loaded at the tip by general forces and general moments. The program is capable of analysing from 1 to 360 points in each of 1 to 5 cross sections of the pipe. The points have to be at the same radial distance from the center of the pipe, and separated from one another by angular intervals which are submultiples of 360° , from 1° to 360° .

The data deck, see Fig. B.1, includes the following cards:

Card No. 1. Format 3F20.8

- a. External diameter of the pipe, inches.
- b. Internal diameter of the pipe, inches.
- c. Pipe length, inches.

Card No. 2. Format 3E20.8, 2F10.8

- a. Young's modulus of elasticity, psi.
- b. Shear modulus of elasticity, psi. This can be omitted in which case it will be computed by the program from $E/2(1 + v)$.
- c. Coefficient of thermal expansion. This can be omitted. It is not actually used in any present application.
- d. Poisson's ratio.
- e. Specific weight of the material, pounds per cubic inch.

Card No. 3. Format 2I5, 5F10.5

- a. Number of points to be analysed in each cross section.
- b. Number of cross sections to be analysed in the pipe.
- c. The distances from the tip of the pipe to the cross sections to be analysed, inches.

Card No. 4. Format F80.2

a. Radial distance of the points to be analysed, inches.

NOTE: for applications to other parts of this thesis, evaluations are made only at the outside surface, but the program is capable of making evaluations at interior points.

Card No. 5. Format F80.2

a. Internal pressure, psi.

Card No. 6. Format 3F20.8

a. Components of the force applied at the tip of the pipe, vertical, sideways and outwards from the tip, so as to form a right handed set of orthogonal axes, lbs.

Card No. 7. Format 3F20.8

a. Components of the moment applied at the tip of the pipe, in the same arrangement as for the forces, lb-inches.

Card No. 8. Format F80.2

a. This is a sentinel card with one number on it. A negative number indicates that the job is finished; a zero or any positive number indicates that the program will be rerun for another set of radial distance and loads, in which case, cards similar to No. 4 through 8 must follow this one.

OUTPUTS (all arguments are outputs)

- RO. External radius of the pipe.
- RI. Internal radius of the pipe.
- XE. Young's modulus of elasticity.

G. Shear modulus of elasticity.

XNU. Poisson's ratio.

SPECW. Specific weight of the material.

D. A constant evaluated when analysing shear stresses produced by shearing forces.

XL1. A one dimensional array containing the distances from the tip to each cross section considered.

NPOINT. Number of points analysed in each cross section.

AREA. Cross sectional area of the pipe.

ALL1. A three dimensional array intended to store the stresses at any of the points, in case the points are at the external surface of the pipe; otherwise, it has no meaning.

Other subroutines which are needed and listings for which are given in this Appendix are:

TORS: used by CANTIL to compute shear stresses produced by torsional moments.

BEND: used by CANTIL to compute bending stresses.

AXIAL: used by CANTIL to compute stresses due to axial forces.

SHEAR: used by CANTIL to compute shear stresses produced by shear forces.

CROSS: computes the cross product of two vectors.

DOT: computes the dot product of two vectors.

TIMES: computes the product of a scalar and a vector.

PLUS: computes the sum of two vectors.

MINUS: computes the difference between two vectors.

TRANSF: computes the components of a vector in another angularly displaced set of axes.

METHOD.

The vectorial approach discussed in reference 5 of this thesis is used to compute stresses produced at points of the cross section by internal pressure and forces and moments acting at the centroid of that cross section.

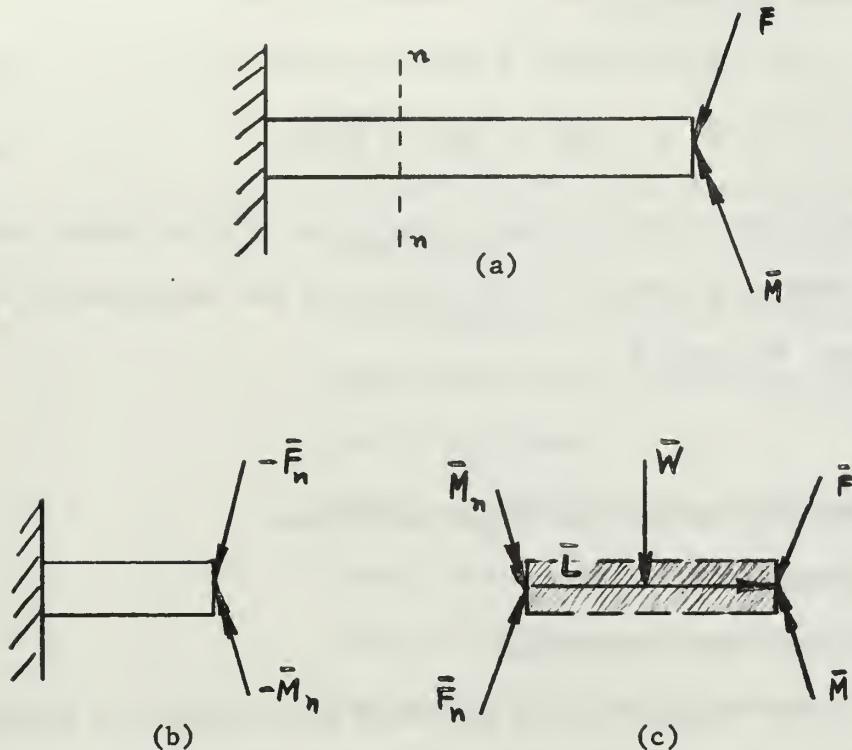


Fig. A-2.

Those forces and moments, in turn, are computed by the classical vectorial analysis of the statics of a beam. If the force \bar{F} and the moment \bar{M} are applied at one end of a beam, Fig. A.2.a., the force \bar{F}_n and moment \bar{M}_n acting at the centroid of the cross section nn, Fig. A.2.c., are found by considering the equilibrium of the segment of beam from the tip to the cross section. Summing forces and taking moments about the centroid of the cross section

$$\bar{F} + \bar{W} + \bar{F}_n = 0 \quad (\text{A.1})$$

$$\bar{M}_n + \bar{M} + \bar{Lx}\bar{F} + \frac{1}{2}\bar{Lx}\bar{W} = 0 \quad (\text{A.2})$$

where \bar{L} is the vector distance from the centroid of the cross section to the point of application of the loads at the tip, and \bar{W} is the force due to the weight of the segment of beam.

Solving for \bar{F}_n and \bar{M}_n

$$\bar{F}_n = -(\bar{F} + \bar{W}) \quad (\text{A.3})$$

$$\bar{M}_n = -(\bar{M} + \bar{Lx}\bar{F} + \frac{1}{2}\bar{Lx}\bar{W}) \quad (\text{A.4})$$

The forces and moments acting at the cross section under consideration on the remaining portion of the beam, are the negatives of \bar{F}_n and \bar{M}_n found above, Fig. A.2.b.

PRINT-OUTS

The print-outs are several pages as follows

1st page.

- a. Pipe dimensions
- b. Computed cross sectional area, moment of inertia and polar moment of inertia of the cross section.
- c. Material properties.
- d. Number of points analysed in each cross section, number of cross sections analysed and their distances from the tip.

2nd page.

- a. Radial distance of the points analysed .
- b. Loads applied to the tip.
- c. Distance from the tip to the first cross section analysed.

d. Computed loads acting at that cross section

Following pages. For each analysed point the print-out contains:

- a. Angular position of the point in the cross section (degrees)
- b. Forces and moments acting at that cross section in radial, tangential and axial components.
- c. Stresses produced by individual loads.

SHTZT is the shear stress in the ZT direction produced by the axial component of the moment
SBZZ axial (ZZ direction) stress produced by the tangential component of the moment.

SAZZ axial stress produced by the axial component of the force.

SHFZR and SHFZT are stresses produced by the radial and tangential components of the force.

SPRR and SPTT are radial and tangential stresses produced by the internal pressure.

- d. If the analysed points are in the external surface of the pipe, there will be additional print-outs of the total axial, tangential and shear stresses at the point.

Analysis of other cross sections for the initial and/or other radial distance and tip loads and pressure produce similar outputs as described from page 2 on. The complete program and sample outputs of the analysis made for points at the external surface and $\frac{1}{2}$ in. under that surface of cross sections 1 and 2 of the pipe shown in Fig. A.1., follow this page.

```

C SUBROUTINE CANTIL(RO,RI,XE,XNU,SPECW,D,XL1,NPOINT,AREA,ALL1)
C TO FINISH THE STATE OF STRESS AT ANY POINT IN A CANTILEVERED STRAIGHT
C PIPE OF CONCENTRIC BORE, LOADED AT THE TIP BY GENERAL FORCES AND
C MOMENTS AND WITH INTERNAL PRESSURE.
C
C DIMENSION AF(3),AXM(3),F(3),XMM(3),R(3),ER(3),EZ(3),
1 SHFTZ(3),SBZZ(3),SAZZ(3),SHFZR(3),SHFZT(3),SPRR(3),SPTT(3),
1 DIMENSION E(3),A1(3),A2(3),A3(3),A4(3),XL1(5),XL2(3),
1 AXM(3),ALL1(3,366,5),AF1(3),FWIGHT(3)
C
C PRINT 500
500 FORMAT(1H1)
C PIPE DIMENSIONS. DO, DI, AND XL ARE EXTERNAL AND INTERNAL DIAMETERS
C AND PIPE LENGTH RESPECTIVELY.
C
C READ 50, DO, DI, XL
50 FORMAT(3F20.8)
C WRITE(6,100)
100 FORMAT(15X,20HINTERNAL DIAMETER =, F8.2, 7H INCHES /
15X,20HINTERNAL DIAMETER =, F8.2, 7H INCHES //,
15X,20LENGTH =, F8.2, 7H INCHES // )
C
C RO = DO/2.0
RO = DI/2.0
RO2 = RO*RO
RI2 = RI*RI
PI = 3.14159265
ARFA = PI*(RO2 - RI2)
ERTIA = AREA*(RO2 + RI2)/4.0
POLER = 2.0*ERTIA
C
C WRITE(6, 103) AREA, ERTIA, POLER
103 FORMAT(15X,20HCROSS SECTIONAL AREA =, F12.4, 4H IN2 /
15X,20HMOMENT OF INERTIA =, F12.4, 4H IN4 // )
C
C MATERIAL PROPERTIES. XE IS MODULUS OF ELASTICITY. G IS SHEAR MODULUS
C OF ELASTICITY WHICH WILL BE COMPUTED IF NOT GIVEN. ALPHA IS COEFF. OF
C THERMAL EXPANSION IF NOT GIVEN. A PRINTOUT WILL SAY SO. XNU IS
C POISSON RATIO. SPFCW IS SPECIFIC WEIGHT.

```

```

C      READ 51, XE, G, ALPHA, XNU, SPFCW
C      FORMAT(3E2.8, 2HMODULUS OF ELASTICITY, 3X, E10.2, 4H PSI )
C
C 51      PRINT 200, XF
C          FORMAT(15X, 21HMODULUS OF ELASTICITY, 3X, E10.2, 4H PSI )
C
C 200      IF ( G ) 3, 4, 5
C          G = XE / ( 2. ) * ( 1.0 + XNU )
C
C 3      PRINT 201, G
C          FORMAT(15X, 27HSHFAR MODULUS OF ELASTICITY, 3X, E10.2, 4H PSI )
C
C 4      IF ( ALPHA ) 5, 6, 7
C          PRINT 202, G
C          FORMAT(15X, 42HCOEFFICIENT OF THERMAL EXPANSION NOT GIVEN )
C
C 5      GO TO 7
C
C 6      PRINT 203, ALPHA
C          FORMAT(15X, 32HCOEFFICIENT OF THERMAL EXPANSION, 3X, E10.2,
C          1 13H IN/IN/DEGREE )
C
C 7      WRITE(6,204) XNU, SPFCW
C          FORMAT(15X, 13HPOISSON RATIO, 3X, F6.3 /
C          1 15X, 15HSPECIFIC WEIGHT, 3X, F8.4, 7H LB/IN3 // )
C
C 203      C NPOINT IS THE NUMBER OF POINTS TO BE ANALYSED IN EACH CROSS SECTION.
C 204      C NSECT IS THE NUMBER OF CROSS SECTIONS TO BE ANALYSED. XL1 IS ONE
C          C DIMENSIONAL ARRAY CONTAINING THE DISTANCES OF THE CROSS SECTIONS FROM
C          C THE TIP.
C
C 205      READ 54, NPOINT, NSECT, XL1
C          -54      FORMAT(2I5, 5F10.5)
C
C 206      WRITE(6,700) NPOINT, NSECT, XL1, 15 /
C          700      FORMAT(15X, 15X, 2SHNUMBER OF POINTS TO ANALYSE, 15 /
C          1 15X, 15X, 15 / DISTANCES FROM THE TIP TO THE CROSS SECTIONS, INCHES)
C          2 3 1, // 20X, 5F8.3 )
C
C 207      C UNIT VECTORS IN POLAR DIRECTIONS
C
C 208      ER(1) = 1.0
C          ER(2) = 0.0
C          ER(3) = 0.0
C          ET(1) = 0.0
C          ET(2) = 1.0
C
C 209      CANT0440
C          CANT0450
C          CANT0460
C          CANT0470
C          CANT0480
C          CANT0490
C          CANT0500
C          CANT0510
C          CANT0520
C          CANT0530
C          CANT0540
C          CANT0550
C          CANT0560
C          CANT0570
C          CANT0580
C          CANT0590
C          CANT0600
C          CANT0610
C          CANT0620
C          CANT0630
C          CANT0640
C          CANT0650
C          CANT0660
C          CANT0670
C          CANT0680
C          CANT0690
C          CANT0700
C          CANT0710
C          CANT0720
C          CANT0730
C          CANT0740
C          CANT0750
C          CANT0760
C          CANT0770
C          CANT0780
C          CANT0790
C          CANT0800
C          CANT0810
C          CANT0820
C          CANT0830
C          CANT0840
C          CANT0850
C          CANT0860

```

```

C
ET(3) = 0.0
EZ(1) = 0.0
EZ(2) = 0.0
EZ(3) = 1.0

C DELTA IS THE ANGULAR INTERVAL IN DEGREES BETWEEN POINTS. R IS THE POS-
C ITION VECTOR OF THE POINT TO BE ANALYSED TAKING AS CENTER THE CENTER
C OF THE CROSS SECTION. XL2 IS THE DISTANCE VECTOR FROM THE CENTER OF
C THE CROSS SECTION TO THE TIP.

C DELTA = 360/NPOINT
THETA2 = PI/180.0

C      8   READ 900, R(1)
900  FORMAT( F80.2 )
      R(2) = 0.0
      R(3) = 0.0
      XL2(1) = 0.0
      XL2(2) = 0.0

C LOADS APPLIED IN VERTICAL DOWNTOWARDS (1), TRANSVERSE (2) AND AXIALLY
C OUTWARDS FROM THE TIP (3) DIRECTIONS. P IS THE INTERNAL PRESSURE AT
C AFL1 AND AXM1 ARE THE GENERAL FORCE AND GENERAL MOMENT ACTING AT THE
C TIP.

C      READ(5, 52) AFL1, AXM1
52   FORMAT( F80.2, AF1, AXM1
      52   FORMAT( F20.8, 3F20.8 )

C      DO 11 J = 1, NSECT
C
C      PRINT 500
C      PRINT 901, R(1)
901  FORMAT( /15X, *RADIAL DISTANCE OF POINTS =', F12.5, ' INCHES' // )
C
C      PRINT 800
C      FORMAT(15X, 24HLOADS APPLIED TO THE TIP, / )
800  WRITE(6, 302) P, AFL1, AXM1
      302 FORMAT( 15X, 20HINTERNAL PRESSURE =', F12.2, ' 4H PSI /
      1           15X, 20HAPPLIED FORCE =', 3F12.2, ' 3H LB /
      2           15X, 20HAPPLIED MOMENT =', 3F12.2, ' 6H LB-IN // )

```

```

C      XL2(3) = XL1(J)
C      PRINT 801
C      FORMAT(15X,'DISTANCE FROM THE TIP TO THE CROSS SECTION',F10.2,
C      1, INCHES,')
C
C      PRINT 801
C      FORMAT(//15X,'LOADS ACTING ON THE CROSS SECTION')
C
C      FWGHT IS THE FORCE DUE TO THE WEIGHT OF THE PIPE FROM THE TIP TO THE
C      CROSS SECTION. AF AND AXM ARE THE LOADS ACTING ON THE CROSS SECTION.
C
C      FWGHT(1) = AREA*SPECW*XL2(3)
C      FWGHT(2) = 0.0
C      FWGHT(3) = 0.0
C      CALL PLUS( FWGHT, AF1, AF )
C      CALL CROSS( XL2, AF1, AI )
C      CALL PLUS( AXM1, AI, AXM )
C      CALL TIMES( 0.5, XL2, A2 )
C      CALL CROSS( A2, FWGHT, A3 )
C      CALL PLUS( AXM, A3, AXM )
C
C      WRITE(6, 302) P, AF, AXM
C
C      INDEX = 1
C      WRITE(6, 500)
C
C      DO 1 I = 1, NPOINT
C      AI = I
C
C      IF( INDEX .LE. 4 ) GO TO 25
C      INDEX = 1
C      WRITE(6, 500)
C      INDEX = INDEX + 1
C
C      THETA1 IS THE ANGLE IN DEGREES FROM THE DOWNWARDS DIRECTION TO THE
C      POINT TO BE ANALYSED, IN THE POSITIVE CCW DIRECTION. F AND XM ARE
C      THE FORCE AND MOMENT ACTING ON THE CROSS SECTION, WITH COMPONENTS IN
C      THE POLAR DIRECTIONS.
C
C      THETA1 = (AI - 1.0)*DELT
C      PRINT 21
C      FORMAT( //15X, 7THHTA =, FR.2, 3X, THDEGRES )
C

```

C C C C C C C C C C C C C C C

C THETIA = THETA1*THETA2

C CALL TRANSF(AF, THETA, XM)
C CALL TRANSF(AX1, THFTA, XM)

C 620 1 WRITE(6, 600) F, XM
FORMAT(15X, 2CHTRANSFORMED FORCE =, 3E12.5, 3H LB /
15X, 2E12.5, 6H LB-IN /)

C SHTZT IS THE SHEAR STRESS IN THE ZT DIRECTION PRODUCED BY THE TORSIONAL MOMENT. SBZZ IS THE STRESS IN THE AXIAL ZZ DIRECTION PRODUCED BY THE BENDING MOMENTS. SAZZ IS STRESS IN THE AXIAL ZZ DIRECTION PRODUCED BY THE AXIAL FORCE. SHFZR AND SHFZT ARE THE SHEAR STRESSES PRODUCED IN THE ZR AND ZT DIRECTIONS BY THE SHEAR FORCES. SPRR AND SPRT ARE THE STRESSES IN THE RADIAL RR AND TANGENTIAL TT DIRECTIONS PRODUCED BY THE INTERNAL PRESSURE.

C CALL TORS(XM, R, EZ, ERTIA, SHTZT)
C CALL BEND(XM, R, EZ, FRTIA, SBZZ)

C CALL AXIAL(F, EZ, AREA, SAZZ)

C CALL SHEAR(F, R, ER, ET, ERTIA, XNU, RI2, RC2, SHFZR, SHFZT, D)

C CALL PRESS(P, R, RI2, RO2, SPRR, SPRT)

C 22 1 WRITE(6, 22) SHTZT(2), SBZZ(3), SAZZ(3), SHFZT(1), SHFZR(1),
SPRR(1), SPRT(2)

1 FORMAT(15X, 5HSHTZT, F17.5 /
1 15X, 5HSBZZ, E17.5 /
2 15X, 5HSAZZ, E17.5 /
3 15X, 5SHFZR, E17.5 /
4 15X, 5SHFZT, E17.5 /
5 15X, 5HSPRR, F17.5 /
6 15X, 5HSPRT, F17.5 /

C 1002 IF(R(1) - RO) 1001, 1002, 1001
CONTINUE

C C ALL IS AN ARRAY INTENDED TO MEMORIZE THE STRESSES AT ANY ONE OF THE POINTS IN CASE THOSE POINTS ARE AT THE EXTERNAL SURFACE OF THE PIPE.

CANT1730
CANT1740
CANT1750
CANT1760
CANT1770
CANT1780
CANT1790
CANT1800
CANT1810
CANT1820
CANT1830
CANT1840
CANT1850
CANT1860
CANT1870
CANT1880
CANT1890
CANT1900
CANT1910
CANT1920
CANT1930
CANT1940
CANT1950
CANT1960
CANT1970
CANT1980
CANT1990
CANT2000
CANT2010
CANT2020
CANT2030
CANT2040
CANT2050
CANT2060
CANT2070
CANT2080
CANT2090
CANT2100
CANT2110
CANT2120
CANT2140
CANT2150

```

C OTHERWISE; IT HAS NO MEANING.
C ALL(1...) ARE STRESSES IN THE TANGENTIAL DIRECTION.
C ALL(2...) ARE STRESSES IN THE AXIAL Z DIRECTION.
C ALL(3...) ARE SHEAR STRESSES IN THE Z DIRECTION.

C
C      ALL(1,I,J) = SPIT(2)
C      ALL(2,I,J) = SBZZ(3) + SAZZ(3)
C      ALL(3,I,J) = SHFT(2) + SHFT(2)

C
C      PRINT 31
C      FORMAT(/ 27X, 13HTT DIRECTION, 4X, 13HZZ DIRECTION, 4X, 13HZT
C      DIRECTION )
C
C      PRINT 32 ( K, I, J, K = 1, 3 )
C      FORMAT( 23X, 3( 5HALL(, II, 1H,, 12, 1H,, II, 1H), 5X) )

C      PRINT 33 ( ALL(1, K, I, J ), K = 16..5, 2E17.5 )
C      FORMAT( 15X, 4, STRESSES, J, E16..5, 2E17.5 )

C 1001 CONTINUE
C 1 CONTINUE
C 11 CONTINUE

C
C FIN IS A SENTINEL CARD WHEN NOT NEGATIVE ( MAY BE A BLANK CARD ) THEN
C OTHER SET OF RADIAL DISTANCE AND TIP LOADS WILL BE READ AND THE
C PROGRAM WILL RERUN. IF FIN IS A NEGATIVE NUMBER, THE PROGRAM WILL
C PROCEED TO THE NEXT STEP.

C
C      READ 501, FIN
C      FORMAT( F80.5 )
C
C      IF ( FIN ) 999, 8, 8
C      CONTINUE
C      RETURN
C      END

```

```

C SUBROUTINE TORS(XM,R,EZ,ERTIA,STZT)
C TO FIND THE SHEAR STRESS PRODUCED BY THE TORSIONAL COMPONENT OF THE
C APPLIED MOMENT XM•R IS THE POSITION VECTOR OF THE POINT TO BE ANALYZED
C EZ IS UNIT VECTOR IN THE AXIAL DIRECTION. ERTIA IS THE MOMENT OF IN-
C FERTIA. STZT IS THE PRODUCED SHEAR STRESS IN THE Z1 DIRECTION.
C
C DIMENSION XM(3), R(3), A1(3), EZ(3), STZT(3)
C CALL CROSS(EZ,R,A1)
C CALL DOT(XM,EZ,Z)
C CALL TIMES(Z/(2.*ERTIA),A1,STZT)
C RETURN
C END

```

TORS0010
 TORS0020
 TORS0030
 TORS0040
 TORS0050
 TORS0060
 TORS0070
 TORS0080
 TORS0090
 TORS0100
 TORS0110
 TORS0120
 TORS0130

```

C SUBROUTINE BEND(XM, R, EZ, FRTIA, SBZZ, )
C TO FIND THE STRESSES PRODUCED BY THE BEARING COMPONENT OF THE APPLIED
C MOMENT XM. R IS THE POSITION VECTOR OF THE POINT TO BE ANALYZED. EZ
C IS UNIT VECTOR IN THE AXIAL DIRECTION; FRTIA IS MOMENT OF INERTIA.
C SBZZ IS STRESS PRODUCED IN THE AXIAL ZZ DIRECTION.
C
C DIMENSION XM(3), R(3), EZ(3), A1(3), SBZZ(3)
C CALL CROSS(R, EZ, A1)
C CALL DOT(A1, XM, Z)
C CALL TIMES(Z/FRTIA, EZ, SBZZ)
C RETURN
END

```

BEND0010
BEND0020
BEND0030
BEND0040
BEND0050
BEND0060
BEND0070
BEND0080
BEND0090
BEND0100
BEND0110
BEND0120
BEND0130

```

C SUBROUTINE AXIAL( F, EZ, AREA, SAZZ )
C TO FIND THE STRESSES PRODUCED BY THE AXIAL COMPONENT OF THE APPLIED
C FORCE F. F IS UNIT VECTOR IN THE AXIAL DIRECTION. AREA IS THE AREA
C OF THE CROSS SECTION. SAZZ IS THE STRESS PRODUCED IN THE AXIAL ZZ
C DIRECTION
C
C DIMENSION F(3), EZ(3), A1(3), SAZZ(3)
C CALL DOT( F, EZ, ZZ )
C CALL TIMES( Z/AREA, EZ, SAZZ )
C RETURN
C END
AXIA0010
AXIA0020
AXIA0030
AXIA0040
AXIA0050
AXIA0060
AXIA0070
AXIA0080
AXIA0090
AXIA0100
AXIA0110
AXIA0120

```

```

C SUBROUTINE SHFAR(F,R,ER,ET,ERTIA,XNU,RI2,RO2,SHFRZ,SHFTZ, D )
C TO FIND THE SHEAR PRODUCED BY THE TRANSVERSE COMPONENTS OF THE APPLIED FORCE F. R IS POSITION VECTOR OF THE POINT TO BE ANALYZED. ER AND ET ARE UNIT VECTORS IN THE RADIAL AND TANGENTIAL DIRECTIONS. ERTIA IS MOMENT OF INERTIA OF THE CROSS SECTION. XNU IS POISSON'S RATIO. RI2 AND RO2 ARE THE SQUARE OF THE INTERNAL AND EXTERNAL RADII OF THE CROSS SECTION. SHFRZ AND SHFTZ ARE THE SHEAR STRESSES PRODUCED IN THE RZ AND TZ DIRECTIONS. D IS A PARAMETER OUTPUT TO BE USED SOMEWHERE ELSE.
C
C DIMENSION F(3),R(3),ER(3),ET(3),SHFRZ(3),SHFTZ(3)
C CALL DOT( R, P, R2 )
C ENE = XNU + 1.5
C ALPHA = ENE*( RI2 + RO2 ) - R2
C BETA = ENE*( R2 + RI2*RO2/R2 ) - R2
C Z1 = 4.0*ERTIA*( 1.0 + XNU )
C C = ( BETA - ALPH )/Z1
C D = ( BETA + ALPH )/Z1
C CALL DOT( F, ER, Z )
C CALL TIMES( -C*Z, ER, SHFRZ )
C CALL DOT( F, ET, Z )
C CALL TIMES( D*Z, ET, SHFTZ )
C RETURN
C
C SHEA0010
C SHEA0020
C SHEA0030
C SHEA0040
C SHEA0050
C SHEA0060
C SHEA0070
C SHEA0080
C SHEA0090
C SHEA0100
C SHEA0110
C SHEA0120
C SHEA0130
C SHEA0140
C SHEA0150
C SHEA0160
C SHEA0170
C SHEA0180
C SHEA0190
C SHEA0200
C SHEA0210
C SHEA0220
C SHEA0230
C SHEA0240
C SHEA0250

```

```

C SUBROUTINE PRESS(P, R, RI2, R02, SPRR, SPTT )
C TO FIND THE STRESSES IN THE RADIAL AND TANGENTIAL DIRECTIONS PRODUCED
C BY THE INTERNAL PRESSURE P. R IS THE POSITION VECTOR OF THE POINT TO
C BE ANALYZED. RI2 AND R02 ARE THE RADIAL AND EXTERNAL
C RADIUS OF THE CROSS SECTION. SPRR AND SPTT ARE THE STRESSES PRODUCED
C IN THE RADIAL AND TANGENTIAL DIRECTIONS.
C
C DIMENSION SPRF(3), SPTT(3), R(3)
CALL DOT( R(1)*R(1), R(2)*R(2) )
Z1 = P*RI2/(R02 - RI2 )
SPRR(1) = Z1*( 1.0 - Z1 )
SPTT(2) = Z1*( 1.0 + Z1 )
SPRR(2) = 0.0
SPRR(3) = 0.0
SPTT(1) = 0.0
SPTT(3) = 0.0
RETURN
END

```

```

C      SUBROUTINE CROSS( A, B, C )
C      VECTOR PRODUCT OF VECTORS A AND B RESULTING VECTOR C.
C
C      DIMENSION A(3),B(3),C(3)
C      C(1) = A(2)*B(3) - A(3)*B(2)
C      C(2) = -A(1)*B(3) + A(3)*B(1)
C      C(3) = A(1)*B(2) - A(2)*B(1)
C      RETURN
C      END

```

```
C SUBROUTINE DOT( A, B, C )
C SCALAR PRODUCT OF VECTORS A AND B RESULTING C.
C
C DIMENSION A(3),B(3)
C      = A(1)*B(1) +A(2)*B(2) + A(3)*B(3)
C      RETURN
C      END
C
C          DOT 0010
C          DOT 0020
C          DOT 0030
C          DOT 0040
C          DOT 0050
C          DOT 0060
C          DOT 0070
C          DOT 0080
```

C SUBROUTINE TIMES(A, B, C)
C MULTIPLICATION OF THE SCALAR A WITH THE VECTOR B RESULTING VECTOR C.
C
DIMENSION B(3), C(3)
DO 1 J = 1,3
1 C(J) = A*B(J)
RETURN
END

TIME0010
TIME0020
TIME0030
TIME0040
TIME0050
TIME0060
TIME0070
TIME0080
TIME0090

```
C SUBROUTINE PLUS( A, B, C )
C ADD VECTORS A AND B RESULTING C.
C
C DIMENSION A(3), B(3), C(3)
DO 1 J=1,3
1   C(J) = A(J) + B(J)
      RETURN
END
```

```
PLUS0010
PLUS0020
PLUS0030
PLUS0040
PLUS0050
PLUS0060
PLUS0070
PLUS0080
PLUS0090
```

C SUBROUTINE MINUS(A, B, C)
C SUBSTRACT VECTOR B FROM VECTOR A RESULTING VECTOR C.
DIMENSION A(3), B(3), C(3)
DO 1 J = 1, 3
C(J) = A(J) - B(J)
1 RETURN
END

MINU0010
MINU0020
MINU0030
MINU0040
MINU0050
MINU0060
MINU0070
MINU0080
MINU0090

```

C      SUBROUTINE TRANSF(A, THETA, C )
C      TO FIND COMPONENTS OF VECTOR A IN OTHER AXIS ROTATED BY AN ANGLE
C      A IS CALLED C IN THE NEW AXIS.
C
DIMENSION A(3), E(3,3), C(3)
E(1,1) = COS(THETA)
E(2,1) = SIN(THETA)
E(3,1) = 0.0
E(1,2) = -SIN(THETA)
E(2,2) = COS(THETA)
E(3,2) = 0.0
E(1,3) = 0.0
E(2,3) = 0.0
E(3,3) = 1.0
DO 1 I = 1,3
C(I) = 0.0
DO 1 J = 1,3
C(I) = C(I) + A(J)*E(J,I)
1      RETURN
END

```

EXTERNAL DIAMETER = 12.00 INCHES
INTERNAL DIAMETER = 10.00 INCHES
LENGTH = 90.00 INCHES

CROSSSECTIONAL AREA = 34.5575 IN²
MOMENT OF INERTIA = 527.0020 IN⁴
POLAR MOMENT OF I. = 1054.0039 IN⁴

MODULUS OF ELASTICITY 0.30E 08 PSI
SHEAR MODULUS OF ELASTICITY 0.12E 08 PSI
COEFFICIENT OF THERMAL EXPANSION NOT GIVEN
POISSON RATIO 0.300
SPECIFIC WEIGHT 0.2830 LB/IN³

NUMBER OF POINTS TO ANALYSE 12
NUMBER OF SECTIONS TO ANALYZE 2
DISTANCES FROM THE TIP TO THE CROSS SECTIONS, INCHES
30.000 60.000 0.0 0.0 0.0

RADIAL DISTANCE OF POINTS = 6.00000 INCHES

LOADS APPLIED TO THE TIP

INTERNAL PRESSURE	=	100.00	PSI		
APPLIED FORCE	=	100.00		100.00	100.00 LB
APPLIED MOMENT	=	100.00		100.00	100.00 LB-IN

DISTANCE FROM THE TIP TO THE CROSS SECTION 30.00 INCHES

LOADS ACTING ON THE CROSS SECTION

INTERNAL PRESSURE	=	100.00	PSI		
APPLIED FORCE	=	393.39		100.00	100.00 LB
APPLIED MOMENT	=	-2900.00		7500.89	100.00 LB-IN

THETA = 0.0 DEGREES
 TRANSFORMED FORCE = 0.39339E 03 0.10000E 03 0.10000E 03 LB
 TRANSFORMED MOMENT = -0.29000E 04 0.75000E 04 0.10000E 03 LB-IN

SHTZT 0.56926E 00
 S_BZZ -0.35399E 02
 S_AZZ 0.28937E 01
 SHEZR 0.0
 SHEZT 0.53861E 01
 S_PER 0.0
 S_PTT 0.45455E 03

	TIT DIRECTION ALL1(1, 1, 1)	ZIZ DIRECTION ALL1(2, 1, 1)	ZIT DIRECTION ALL1(3, 1, 1)
STRESSES	0.45455E 03	-0.82505E 02	0.59553E 01

THETA = 30.00 DEGREES
 TRANSFORMED FORCE = 0.39069E 03 -0.11029E 03 0.10000E 03 LB
 TRANSFORMED MOMENT = 0.12390E 04 0.79460E 04 0.10000E 03 LB-IN

SHTZT 0.56926E 00
 S_BZZ -0.20466E 02
 S_AZZ 0.28937E 01
 SHEZR 0.0
 SHEZT 0.59297E 01
 S_PER 0.0
 S_PTT 0.45455E 03

	TIT DIRECTION ALL1(1, 2, 1)	ZIZ DIRECTION ALL1(2, 2, 1)	ZIT DIRECTION ALL1(3, 2, 1)
STRESSES	0.45455E 03	-0.87572E 02	-0.53605E 01

THETA = 60.00 DEGREES
 TRANSFORMED FORCE = 0.28330E 03 -0.29069E 03 0.10000E 03 LB
 TRANSFORMED MOMENT = 0.50460E 04 0.62619E 04 0.10000E 03 LB-IN

SHTZT 0.56926E 00
 S_BZZ -0.71293E 02
 S_AZZ 0.28937E 01
 SHEZR 0.0
 SHEZT -0.15657E 02
 S_PER 0.0
 S_PTT 0.45455E 03

	TIT DIRECTION ALL1(1, 3, 1)	ZIZ DIRECTION ALL1(2, 3, 1)	ZIT DIRECTION ALL1(3, 3, 1)
STRESSES	0.45455E 03	-0.68399E 02	-0.15087E 02

THETA = 90.00 DEGREES
 TRANSFORMED FORCE = 0.10000E 03 -0.32339E 03 0.10000E 03 LB
 TRANSFORMED MOMENT = 0.75009E 04 0.29000E 04 0.10000E 03 LB-IN

SHTZT 0.56926E 00
 S_BZZ -0.33017E 02
 S_AZZ 0.28937E 01
 SHEZR 0.0
 SHEZT -0.21183E 02
 S_PER 0.0
 S_PTT 0.45455E 03

	TIT DIRECTION ALL1(1, 4, 1)	ZIZ DIRECTION ALL1(2, 4, 1)	ZIT DIRECTION ALL1(3, 4, 1)
STRESSES	0.45455E 03	-0.30123E 02	-0.20619E 02

THETA = 120.00 DEGREES
TRANSFORMED FORCE = -0.11009E 03-0.39069E 03 0.10000E 03 LB
TRANSFORMED MOMENT = 0.79460E 04-0.12390E 04 0.10000E 03 LB-IN

SHTZT 0.56926E 00
S BZZ 0.14106E 02
S AZZ 0.28937E 01
SHFZR 0.0
SHEZT -0.21043E 02
SPRR 0.0
S PTT 0.45455E 03

STRESSES T'T DIRECTION Z'Z DIRECTION Z'T DIRECTION
ALL1(1, 5,1) ALL1(2, 5,1) ALL1(3, 5,1)
0.45455E 03 0.16999E 02 -0.20473E 02

THETA = 150.00 DEGREES
TRANSFORMED FORCE = -0.29069E 03-0.28330E 03 0.10000E 03 LB
TRANSFORMED MOMENT = 0.62619E 04-0.50460E 04 0.10000E 03 LB-IN

SHTZT 0.56926E 00
S BZZ 0.57449E 02
S AZZ 0.28937E 01
SHFZR 0.0
SHEZT -0.15259E 02
SPRR 0.0
S PTT 0.45455E 03

STRESSES T'T DIRECTION Z'Z DIRECTION Z'T DIRECTION
ALL1(1, 6,1) ALL1(2, 6,1) ALL1(3, 6,1)
0.45455E 03 0.60343E 02 -0.14689E 02

THETA = 180.00 DEGREES
TRANSFORMED FORCE = -0.39339E 03-0.10000E 03 0.10000E 03 LB
TRANSFORMED MOMENT = 0.29000E 04-0.75009E 04 0.10000E 03 LB-IN

SHTZT 0.55926E 00
S BZZ 0.85399E 02
S AZZ 0.23937E 01
SHFZR 0.0
SHEZT -0.53861E 01
SPRR 0.0
S PTT 0.45455E 03

STRESSES T'T DIRECTION Z'Z DIRECTION Z'T DIRECTION
ALL1(1, 7,1) ALL1(2, 7,1) ALL1(3, 7,1)
0.45455E 03 0.88292E 02 -0.48168E 01

THETA = 210.00 DEGREES
TRANSFORMED FORCE = -0.39069E 03 0.11009E 03 0.10000E 03 LB
TRANSFORMED MOMENT = -0.12390E 04-0.79460E 04 0.10000E 03 LB-IN

SHTZT 0.56926E 00
S BZZ 0.90466E 02
S AZZ 0.28937E 01
SHFZR 0.0
SHEZT 0.59297E 01
SPRR 0.0
S PTT 0.45455E 03

STRESSES T'T DIRECTION Z'Z DIRECTION Z'T DIRECTION
ALL1(1, 8,1) ALL1(2, 8,1) ALL1(3, 8,1)
0.45455E 03 0.93360E 02 0.64989E 01

THETA = 240.00 DEGREES

TRANSFORMED FORCE = -0.28330E 03 0.29069E 03 0.10000E 03 LB
TRANSFORMED MOMENT = -0.50459E 04 -0.62619E 04 0.10000E 03 LB-IN

SHTZT 0.56926E 00
S_BZZ 0.71293E 02
S_AZZ 0.28937E 01
SHFZR 0.0
SHEZT 0.15657E 02
S_PRR 0.0
S_PTT 0.45455E 03

T'T DIRECTION Z'Z DIRECTION Z'T DIRECTION
STRESSES ALL1(1, 9, 1) ALL1(2, 9, 1) ALL1(3, 9, 1)
0.45455E 03 0.74187E 02 0.16226E 02

THETA = 270.00 DEGREES

TRANSFORMED FORCE = 0.10000E 03 0.39339E 03 0.10000E 03 LB
TRANSFORMED MOMENT = -0.75009E 04 -0.29000E 04 0.10000E 03 LB-IN

SHTZT 0.56926E 00
S_BZZ 0.33017E 02
S_AZZ 0.28937E 01
SHFZR 0.0
SHEZT 0.21188E 02
S_PRR 0.0
S_PTT 0.45455E 03

T'T DIRECTION Z'Z DIRECTION Z'T DIRECTION
STRESSES ALL1(1, 10, 1) ALL1(2, 10, 1) ALL1(3, 10, 1)
0.45455E 03 0.35911E 02 0.21758E 02

THETA = 300.00 DEGREES

TRANSFORMED FORCE = 0.11009E 03 0.39069E 03 0.10000E 03 LB
TRANSFORMED MOMENT = -0.79460E 04 0.12390E 04 0.10000E 03 LB-IN

SHTZT 0.56926E 00
S_BZZ -0.14106E 02
S_AZZ 0.28937E 01
SHFZR 0.0
SHEZT 0.21043E 02
S_PRR 0.0
S_PTT 0.45455E 03

T'T DIRECTION Z'Z DIRECTION Z'T DIRECTION
STRESSES ALL1(1, 11, 1) ALL1(2, 11, 1) ALL1(3, 11, 1)
0.45455E 03 -0.11212E 02 0.21612E 02

THETA = 330.00 DEGREES

TRANSFORMED FORCE = 0.29069E 03 0.28330E 03 0.10000E 03 LB
TRANSFORMED MOMENT = -0.62619E 04 0.50459E 04 0.10000E 03 LB-IN

SHTZT 0.56926E 00
S_BZZ -0.57449E 02
S_AZZ 0.28937E 01
SHFZR 0.0
SHEZT 0.15259E 02
S_PRR 0.0
S_PTT 0.45455E 03

T'T DIRECTION Z'Z DIRECTION Z'T DIRECTION
STRESSES ALL1(1, 12, 1) ALL1(2, 12, 1) ALL1(3, 12, 1)
0.45455E 03 -0.54555E 02 0.15828E 02

RADIAL DISTANCE OF POINTS = 6.00000 INCHES

LOADS APPLIED TO THE TIP

INTERNAL PRESSURE	=	100.00	PSI		
APPLIED FORCE	=	100.00		100.00	100.00 LB
APPLIED MOMENT	=	100.00		100.00	100.00 LB-IN

DISTANCE FROM THE TIP TO THE CROSS SECTION 60.00 INCHES

LOADS ACTING ON THE CROSS SECTION

INTERNAL PRESSURE	=	100.00	PSI		
APPLIED FORCE	=	686.79		100.00	100.00 LB
APPLIED MOMENT	=	-5900.00		23703.59	100.00 LB-IN

THETA = 0.0 DEGREES
 TRANSFORMED FORCE = 0.68679E 03 0.10000E 03 0.10000E 03 LB
 TRANSFORMED MOMENT = -0.59000E 04 0.23704E 05 0.10000E 03 LB-IN

SHTZT	0.56926E 00			
S BZZ	-0.26987E 03			
S AZZ	0.28937E 01			
SHFZR	0.0			
SHFZT	0.53861E 01			
S PRR	0.0			
S PTT	0.45455E 03			
STRESSES		T'T DIRECTION ALL1(1, 1, 2) 0.45455E 03	Z'Z DIRECTION ALL1(2, 1, 2) -0.26698E 03	Z'T DIRECTION ALL1(3, 1, 2) 0.59553E 01

THETA = 30.00 DEGREES
 TRANSFORMED FORCE = 0.64477E 03 -0.25679E 03 0.10000E 03 LB
 TRANSFORMED MOMENT = 0.67422E 04 0.23478E 05 0.10000E 03 LB-IN

SHTZT	0.56926E 00			
S BZZ	-0.26730E 03			
S AZZ	0.28937E 01			
SHFZR	0.0			
SHFZT	-0.13831E 02			
S PRR	0.0			
S PTT	0.45455E 03			
STRESSES		T'T DIRECTION ALL1(1, 2, 2) 0.45455E 03	Z'Z DIRECTION ALL1(2, 2, 2) -0.26441E 03	Z'T DIRECTION ALL1(3, 2, 2) -0.13262E 02

THETA = 60.00 DEGREES
 TRANSFORMED FORCE = 0.43000E 03 -0.54477E 03 0.10000E 03 LB
 TRANSFORMED MOMENT = 0.17578E 05 0.16961E 05 0.10000E 03 LB-IN

SHTZT	0.56926E 00			
S BZZ	-0.19311E 03			
S AZZ	0.28937E 01			
SHFZR	0.0			
SHFZT	-0.29342E 02			
S PRR	0.0			
S PTT	0.45455E 03			
STRESSES		T'T DIRECTION ALL1(1, 3, 2) 0.45455E 03	Z'Z DIRECTION ALL1(2, 3, 2) -0.19021E 03	Z'T DIRECTION ALL1(3, 3, 2) -0.28773E 02

THETA = 90.00 DEGREES
 TRANSFORMED FORCE = 0.10000E 03 -0.68679E 03 0.10000E 03 LB
 TRANSFORMED MOMENT = 0.23704E 05 0.59000E 04 0.10000E 03 LB-IN

SHTZT	0.56926E 00			
S BZZ	-0.67173E 02			
S AZZ	0.28937E 01			
SHFZR	0.0			
SHFZT	-0.36991E 02			
S PRR	0.0			
S PTT	-0.45455E 03			
STRESSES		T'T DIRECTION ALL1(1, 4, 2) 0.45455E 03	Z'Z DIRECTION ALL1(2, 4, 2) -0.64279E 02	Z'T DIRECTION ALL1(3, 4, 2) -0.36421E 02

RADIAL DISTANCE OF POINTS = 5.50000 INCHES

LOADS APPLIED TO THE TIP

INTERNAL PRESSURE	=	100.00	PSI		
APPLIED FORCE	=	100.00		100.00	100.00 LB
APPLIED MOMENT	=	100.00		100.00	100.00 LB-IN

DISTANCE FROM THE TIP TO THE CROSS SECTION 30.00 INCHES

LOADS ACTING ON THE CROSS SECTION

INTERNAL PRESSURE	=	100.00	PSI		
APPLIED FORCE	=	393.39		100.00	100.00 LB
APPLIED MOMENT	=	-2900.00		7500.89	100.00 LB-IN

THETA = 120.00 DEGREES

TRANSFORMED FORCE = -0.11009E 03 -0.39069E 03 0.10000E 03 LB
TRANSFORMED MOMENT = 0.79460E 04 -0.12390E 04 0.10000E 03 LB-IN

SHTZT	0.52182E 00
S_BZZ	0.12930E 02
S_AZZ	0.28937E 01
SHEZR	-0.72164E -01
SHEZT	-0.22426E 02
S_PRR	-0.43201E 02
S_PTT	0.49775E 03

THETA = 150.00 DEGREES

TRANSFORMED FORCE = -0.29069E 03 -0.28330E 03 0.10000E 03 LB
TRANSFORMED MOMENT = 0.62619E 04 -0.50460E 04 0.10000E 03 LB-IN

SHTZT	0.52182E 00
S_BZZ	0.52662E 02
S_AZZ	0.28937E 01
SHEZR	-0.19054E 00
SHEZT	-0.16262E 02
S_PRR	-0.43201E 02
S_PTT	0.49775E 03

THETA = 180.00 DEGREES

TRANSFORMED FORCE = -0.39330E 03 -0.10000E 03 0.10000E 03 LB
TRANSFORMED MOMENT = 0.29069E 04 -0.75019E 04 0.10000E 03 LB-IN

SHTZT	0.52182E 00
S_BZZ	0.78282E 02
S_AZZ	0.28937E 01
SHEZR	-0.25786E 00
SHEZT	-0.57462E 01
S_PRR	-0.43201E 02
S_PTT	0.49775E 03

THETA = 210.00 DEGREES

TRANSFORMED FORCE = -0.39069E 03 0.11009E 03 0.10000E 03 LB
TRANSFORMED MOMENT = -0.12390E 04 -0.79460E 04 0.10000E 03 LB-IN

SHTZT	0.52182E 00
S_BZZ	0.82927E 02
S_AZZ	0.28937E 01
SHEZR	-0.22509E 00
SHEZT	-0.63195E 01
S_PRR	-0.43201E 02
S_PTT	0.49775E 03

RADIAL DISTANCE OF POINTS = 5.50000 INCHES

LOADS APPLIED TO THE TIP

INTERNAL PRESSURE	=	100.00	PSI
APPLIED FORCE	=	100.00	100.00
APPLIED MOMENT	=	100.00	100.00
			100.00 LB
			100.00 LB-IN

DISTANCE FROM THE TIP TO THE CROSS SECTION 60.00 INCHES

LOADS ACTING ON THE CROSS SECTION

INTERNAL PRESSURE	=	100.00	PSI
APPLIED FORCE	=	686.79	100.00
APPLIED MOMENT	=	-5900.00	23703.59
			100.00 LB
			100.00 LB-IN

THE TA = 0.0 DEGREES

TRANSFORMED FORCE = 0.68679E 03 0.10000F 03 0.10000E 03 LB
TRANSFORMED MOMENT = -0.59000E 04 0.23704F 05 0.10000E 03 LB-IN

SHTZT 0.52182E 00
S_BZZ -0.24738E 03
S_AZZ 0.28937E 01
SHFZR 0.45018E 00
SHEZT 0.37402E 01
S_PRR -0.43201E 02
S_PTT 0.49775E 03

THE TA = 30.00 DEGREES

TRANSFORMED FORCE = 0.64477E 03 -0.25679F 03 0.10000E 03 LB
TRANSFORMED MOMENT = 0.67422E 04 0.23478F 05 0.10000E 03 LB-IN

SHTZT 0.52182E 00
S_BZZ -0.24502E 03
S_AZZ 0.28937E 01
SHFZR 0.42264E 00
SHEZT -0.14740E 02
S_PRR -0.43201E 02
S_PTT 0.49775E 03

THE TA = 60.00 DEGREES

TRANSFORMED FORCE = 0.43000E 03 -0.54477F 03 0.10000E 03 LB
TRANSFORMED MOMENT = 0.17578E 05 0.16961E 05 0.10000E 03 LB-IN

SHTZT 0.52182E 00
S_BZZ -0.17702E 03
S_AZZ 0.28937E 01
SHFZR 0.23186E 00
SHEZT -0.31271E 02
S_PRR -0.43201E 02
S_PTT 0.49775E 03

THE TA = 90.00 DEGREES

TRANSFORMED FORCE = 0.10000E 03 -0.68679F 03 0.10000E 03 LB
TRANSFORMED MOMENT = 0.23704E 05 0.59000E 04 0.10000E 03 LB-IN

SHTZT 0.52182E 00
S_BZZ -0.61575E 02
S_AZZ 0.28937E 01
SHFZR 0.45549E -01
SHEZT -0.39423E 02
S_PRR -0.43201E 02
S_PTT 0.49775E 03

THETA = 120.00 DEGREES
TRANSFORMED FORCE = -0.25679E 03 -0.64477E 03 0.10000E 03 LB
TRANSFORMED MOMENT = 0.23478E 05 -0.67422E 04 0.10000E 03 LB-IN

SHTZT 0.52182E 00
S_BZZ 0.70364E 02
S_AZZ 0.28937E 01
SHFZR -0.16832E 00
SHFZT -0.37011E 02
S_PRR -0.43201E 02
S_PTT 0.49775E 03

THETA = 150.00 DEGREES
TRANSFORMED FORCE = -0.54477E 03 -0.43000E 03 0.10000E 03 LB
TRANSFORMED MOMENT = 0.16961E 05 -0.17578E 05 0.10000F 03 LB-IN

SHTZT 0.52182E 00
S_BZZ 0.18345E 03
S_AZZ 0.28937E 01
SHFZR -0.35709E 00
SHFZT -0.24682E 02
S_PRR -0.43201E 02
S_PTT 0.49775E 03

THETA = 180.00 DEGREES
TRANSFORMED FORCE = -0.68679E 03 -0.10000F 03 0.10000E 03 LB
TRANSFORMED MOMENT = 0.59000F 04 -0.23704E 05 0.10000F 03 LB-IN

SHTZT 0.52182E 00
S_BZZ 0.24738E 03
S_AZZ 0.28937E 01
SHFZR -0.45018E 00
SHFZT -0.57402E 01
S_PRR -0.43201E 02
S_PTT 0.49775E 03

THETA = 210.00 DEGREES
TRANSFORMED FORCE = -0.64477E 03 -0.25679E 03 0.10000E 03 LB
TRANSFORMED MOMENT = -0.67422E 04 -0.23478E 05 0.10000F 03 LB-IN

SHTZT 0.52182E 00
S_BZZ 0.24502E 03
S_AZZ 0.28937E 01
SHFZR -0.42264E 00
SHFZT -0.14740E 02
S_PRR -0.43201E 02
S_PTT 0.49775E 03

THETA = 240.00 DEGREES
TRANSFORMED FORCE = -0.43000E 03 0.54477E 03 0.10000E 03 LB
TRANSFORMED MOMENT = -0.17578E 05-0.16961E 05 0.10000E 03 LB-IN

SHTZT 0.52182E 00
S BZZ 0.17702E 03
S AZZ 0.28937E 01
SHFZR -0.28186E 00
SHFZT 0.31271E 02
S PRR -0.43201E 02
S PTT 0.49775E 03

THETA = 270.00 DEGREES
TRANSFORMED FORCE = -0.10000E 03 0.68679E 03 0.10000E 03 LB
TRANSFORMED MOMENT = -0.23704E 05-0.59000E 04 0.10000E 03 LB-IN

SHTZT 0.52182E 00
S BZZ 0.61575E 02
S AZZ 0.28937E 01
SHFZR -0.65549E-01
SHFZT 0.39423E 02
S PRR -0.43201E 02
S PTT 0.49775E 03

THETA = 300.00 DEGREES
TRANSFORMED FORCE = 0.25679E 03 0.64477E 03 0.10000E 03 LB
TRANSFORMED MOMENT = -0.23478E 05 0.67422E 04 0.10000E 03 LB-IN

SHTZT 0.52182E 00
S BZZ -0.70364E 02
S AZZ 0.28937E 01
SHFZR 0.16832E 00
SHFZT 0.37011E 02
S PRR -0.43201E 02
S PTT 0.49775E 03

THETA = 330.00 DEGREES
TRANSFORMED FORCE = 0.54477E 03 0.43000E 03 0.10000E 03 LB
TRANSFORMED MOMENT = -0.16961E 05 0.17578E 05 0.10000E 03 LB-IN

SHTZT 0.52182E 00
S BZZ -0.18345E 03
S AZZ 0.28937E 01
SHFZR 0.35709E 00
SHFZT 0.24682E 02
S PRR -0.43201E 02
S PTT 0.49775E 03

APPENDIX B

DIGITAL COMPUTER PROGRAM FOR SECTION 2 AND NUMERICAL RESULTS

The programs presented herein provide a method for applying the theory developed in Section 2 of this thesis. Actual situations can be handled by subroutines ZULIA, GUAYRA, CUMANA and DSIMQ (DSIMQ is presented in Appendix D). Simulated situations require in addition subroutines CANTIL presented in Appendix A, and TIUNA and BLANCA presented in this Appendix. The last of the subroutines mentioned is an adaptation of a subroutine taken from reference 7 of this thesis.

ZULIA is used to specify the required information, such as material properties, number of Fourier coefficients to be computed and description of the arrangement of strain gage elements and their transverse sensitivity factor. Enough self explanatory comment statements have been included which specify the introduction of the information. Print-out statements present the information received for processing. Once the information is properly arranged, subroutine GUAYRA is used to make the required computations.

The sequence in GUAYRA follows closely the theoretical development of the method. It has sufficient comment statements separating blocks of computer processing instructions as to follow the theoretical procedure. Print-out statements present the computed Fourier coefficients.

CUMANA is used by GUAYRA for auxiliary computations.

Use of the subroutines is made by a very brief main program TERESITA. This program merely dimensions the various subroutines and turns control over to subroutine ZULIA. The card arrangement of the source deck is as shown in Fig. B.1.

Some sample results of the application of the program as specified at the end of Section 2, follow the card listings of the programs.

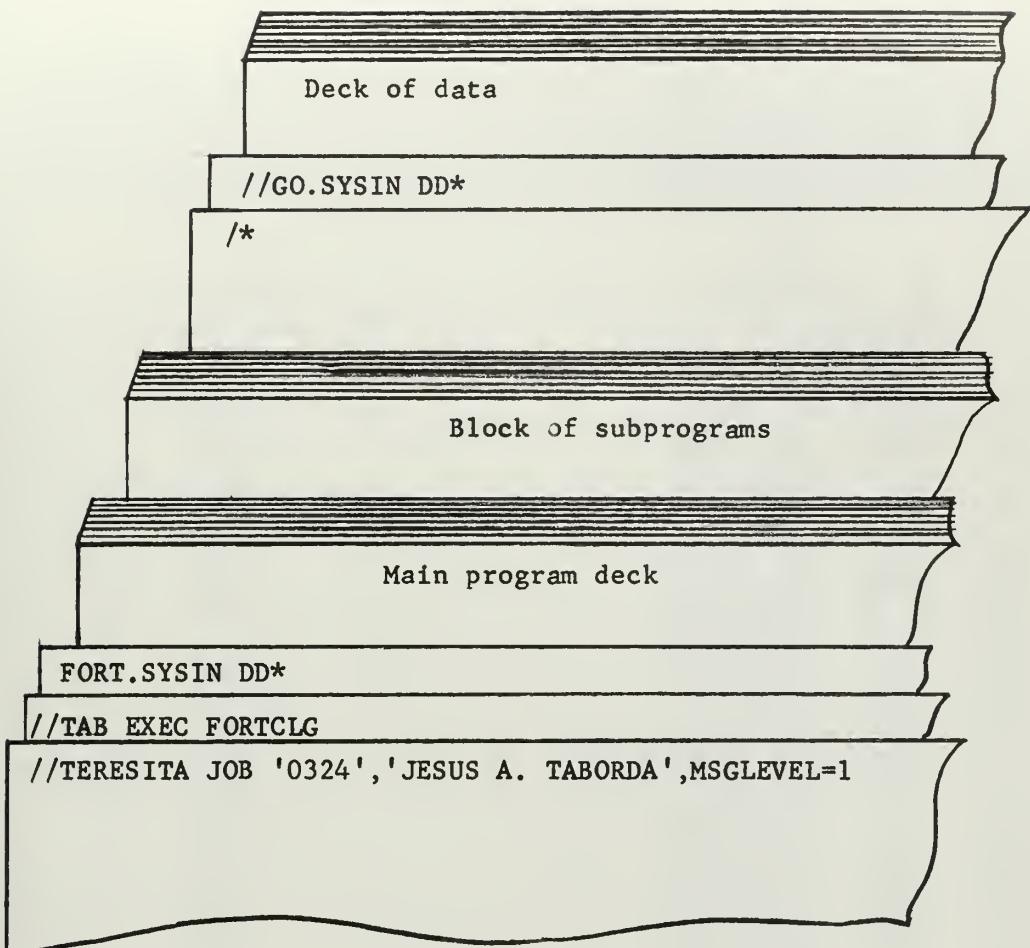


Fig. B.1

```

//TERESILIA JOB '0324', TABORDA, JESUS A., MSGLEVEL=1
//TAB EXEC FORTCLG
//FORT EXEC PGM=IFYFCRT
          00000010
DIMENSION Z1(36),Z2(36),Z3(37),Z4(37),Z5(36,36),Z6(36,36)
1 C(36),ST(36),SL(36),SH(36),STA(10),STB(10),SLA(10),
2 SB(10),SHA(10),SHB(10),
DIMENSION AF1(3),AXM1(3),AF(3),AXM(3),F(3),XM(3),FWGHT(3),
1 XL1(5),XL2(3),ER(3),ET(3),EZ(3),R(3),AL(3),SPT(5),
2 SHFT(3),SB2Z(3),SAZZ(3),SHFZT(3),SHFZR(3),SPRR(3),SPT(3)
DIMENSION F(3,2)
DIMENSION AI(3),A2(3),A3(3),A4(3),F3(3),F4(3),F5(3),F6(3),
1 XM3(3),XM4(3),XM5(3),XM6(3),R1(3),B2(3),B3(3),B4(3),B5(3),
2 CI(36,36),C2(36,36),C3(36,36),C4(36,36),C5(36),C6(36),
C CALL ZULIA
C
C      END

```

SUBROUTINE ZULIA

1 DIMENSION T(36),P(36),R(36),N(36),T1(37),T2(36),NPAD(36),
STRNEX(36),

SPECIFY NUMBER OF ECURIEL COEFFICIENTS NCoeff, NUMBER OF GAGE ELEMENTS NGages, TRANSVERSE SENSITIVITY FACTOR TRANSV, AND CROS-
RESPONDING ANGULAR POSITION IN THE CROSS SECTION (CONTAINED
ARRAY T1) AND ANGULAR ORIENTATION FROM TANGENTIAL (CONTAINED
IN ARRAY P1).

NCoeff = 5
NGages = 26
TRANSV = 0.01
K = 0
DO 5001 I = 1, 36, 3
P1(I) = C*0.0
P1(I+1) = E*0.0
P1(I+2) = 12*0.0
T1(I) = K*30
T1(I+1) = T1(I)
T1(I+2) = T1(I)
5001 K = K + 1

SET ANGLE P1(I) OF GAGE ELEMENT FUNCTIONING IMPROPERLY TO 1.0F6
AND ELIMINATE THOSE NUMBER OF EACH OF THE ELIMINATED ELEMENTS WILL
BE CONTAINED IN ARRAY NPAD FOR PRINT-OUT PURPOSES; ANGULAR POSITION
AND ORIENTATION OF THE REMAINING K ACTIVE ELEMENTS WILL BE CONTAINED
IN ARRAYS T AND P RESPECTIVELY.

P1(12) = 1.0E6
P1(22) = 1.0E6
P1(24) = 1.0E6
P1(25) = 1.0E6
P1(33) = 1.0E6
20 L = 0

DO 3 IF(P1(I) = 1.0E6 .1.0E6) GO TO 34
33 K = K + 1
P(K) = P1(I)
T(K) = T1(I)

```

34 GC TO 3
    L = L + 1
    NBAC(L) = 1
    3 CONTINUE

C SPECIFY MATERIAL PROPERTIES: YOUNG'S MODULUS OF ELASTICITY XE, SHEAR
C MODULUS OF ELASTICITY G, POISSON'S RATIO XNU.
C IF THE MATHEMATICAL MODEL OF A PIPE IS USED, OUT PUTS ARGUMENTS OF
C SUBROUTINE CANTIL WILL DO IT.

    CALL CANTIL(RG,RI,XE,G,XNU,SPECW,D,XL1,NPOINT,ARFA,ALL1)

C SPECIFY READINGS OF THE GAGE ELEMENTS, TO BE CONTAINED IN ARRAY
C STRNEX.

    THEORETICAL READINGS CAN BE OBTAINED FROM THE OUTPUT OF SUBROUTINE
    TIUNA. THIS USES AS INFORMATION THE MATERIAL PROPERTIES AND ANGULAR
    ORIENTATION OF EACH ELEMENT, AND THE STRESS COMPONENTS AT EACH
    ELEMENT CENTER. THIS IS OBTAINED FROM THE OUTPUT ALL1 OF CANTIL
    REQUIRING THE ANGULAR POSITION IN THE CROSS SECTION AND THE CROSS
    SECTION NUMBER TO IDENTIFY THE RIGHT STRESSES.

    NSECT = 1
    CALL TIUNA(XE,G,XNU,NSECT,ALL1,K,T,P,ZZ,TRANSV)

    RANDOM ERRORS CAN BE ADDED TO THE THEORETICAL READINGS OF THE
    GAGES, THE OUTPUT ZZ OF TIUNA, BY USING SUBROUTINE BLANCA.
    SPECIFY THE MULTIPLIER ALT TO OBTAIN (ALT)*(10^7) MAXIMUM RANDOM
    ERROR ADDED.

    ALT = 0.3
    CALL BLANCA(STRNEX,ZZ,K,ALT)

C PRINT-OUT OF ALL THE INFORMATION OBTAINED UP TO NOW.

    500 WRITE(6,500)
        FORMAT(1H1)
    500 WRITE(6,1) NCOEFF
    11  FORMAT(7(/),15X,EXPANSION WITH, I3, FOURIER COEFFICIENTS //)

```

SUBROUTINE JULIA

DIMENSIGN T(36), P(36), R(36), T1(37), T1(37), W(36), A(LI 1(3,360,5), STRNFX(36),

SPECIFY NUMBER OF FOURIER COEFFICIENTS NCOEFF, NUMBER OF GAGE ELEMENTS NGAGES, TRANSVERSE SENSITIVITY FACTOR TRANSV, AND CORRESPONDING ANGULAR POSITION IN THE CROSS SECTION (CONTAINED IN THE ARRAY T) AND ANGULAR ORIENTATION FROM TANGENTIAL AXIS (CONTAINED IN ARRAY P).

NCSEFF = 5
 NGAGES = 36
 TRANSV = 1.C1
 TRANSI = TRANSV*100.0
 K = 0
 CO = 5001
 P1(1) = 1
 P1(1+1) = 1
 P1(1+2) = 2
 T1(1) = 1
 T1(1+1) = 1
 T1(1+2) = 1
 K = K +

SET ANGLE PLANE OF GAGE ELEMENTS FUNCTIONING IMPROPERLY TO 1 OF 6 AND ELIMINATE IT. NUMBER OF EACH OF THE ELIMINATED ELEMENTS WILL BE CONTAINED IN ARRAY NBAD FOR PRINT-OUT PURPOSES. ANGULAR POSITION OF THE REMAINING K ACTIVE ELEMENTS WILL BE CONTAINED

$$3.3 \quad \begin{array}{l} \text{P}_1(K) = P_1(K) + P_1(I) \\ \text{P}(K) = T_1(I) \\ T(K) = T_1(I) \end{array} \quad \begin{array}{l} \text{GE.} \\ \text{1.6E6} \\ 50 \end{array} \quad 34$$

GC TO
34 L = L + 1
 NBAD(L) = 1
 3 CONTINUE

SPECIFY MATERIAL PROPERTIES: YOUNG'S MODULUS OF ELASTICITY XNU, SHEAR MODULUS OF ELASTICITY G, POISSON'S RATIO XNU.
IF THE MATHEMATICAL MODEL OF A PIPE IS USED, PUTS ARGUMENTS OF SUBROUTINE CANTIL WITH IT.

CALL CANTIL(RO, RI, XE, G, XNU, SPECW, D, XL1, NPQINT, ARFA, ALL1)

SPECIFY READINGS OF THE GAGE ELEMENTS, TO BE CONTAINED IN ARRAY STRNEX.

THEORETICAL READINGS CAN BE OBTAINED FROM THE OUTPUT OF SUBROUTINE TIUNA. THIS USES AS INFORMATION THE MATERIAL PROPERTIES AND ANGULAR ORIENTATION OF EACH ELEMENT, AND THE STRESS COMPONENTS AT EACH ELEMENT CENTER. THIS IS OBTAINED FROM THE OUTPUT ALL OF CANTIL REQUIRING THE ANGULAR POSITION IN THE CROSS SECTION AND THE CROSS SECTION NUMBER TO IDENTIFY THE RIGHT STRESSES.

NSECT = 1
CALL TIUNA(XE, G, XNU, NSECT, ALL1, K, T, P, ZZ, TRANSV)

RANDOM ERRORS CAN BE ADDED TO THE THEORETICAL READINGS OF THE GAGES. THE OUTPUT ZZ OF TIUNA, BY USING SUBROUTINE BLANCA, SPECIFY THE MULTIPLIER ALT TO OBTAIN (ALT)*(10⁹) MAXIMUM RANDOM ERROR ADDED.

ALT = 0.3
CALL BLANCA(STRNEX, ZZ, K, ALT)

PRINT-OUT OF ALL THE INFORMATION OBTAINED UP TO NOW.

500 WRITE(6, 500)
 FORMAT(1H1)
 WRITE(6, 11) NCOEFF
11 FORMAT(7(/), 15X, 'EXPANSION WITH', I3, ' FOURIER COEFFICIENTS //')

71

STRUCTURE GUAYA(C,THETA,XNU,NGAGE,PHI,A,B,C,TTRANSV)
 1 JJ,FORMAT,ALLI,TRANSV

GUAYR001
 GUAYR002
 GUAYR003
 GUAYR004
 GUAYR005
 GUAYR006
 GUAYR007
 GUAYR008
 GUAYR009
 GUAYR010
 GUAYR011
 GUAYR012
 GUAYR013
 GUAYR014
 GUAYR015
 GUAYR016
 GUAYR017
 GUAYR018
 GUAYR019
 GUAYR020
 GUAYR021
 GUAYR022
 GUAYR023
 GUAYR024
 GUAYR025
 GUAYR026
 GUAYR027
 GUAYR028
 GUAYR029
 GUAYR030
 GUAYR031
 GUAYR032
 GUAYR033
 GUAYR034
 GUAYR035
 GUAYR036
 GUAYR037
 GUAYR038
 GUAYR039
 GUAYR040
 GUAYR041
 GUAYR042
 GUAYR043
 GUAYR044
 GUAYR045
 GUAYR046
 GUAYR047

DETERMINES FOURIER COEFFICIENTS FOR THE DISTRIBUTIONS OF TANGENTIAL, LONGITUDINAL AND SHEAR STRESSES. ALL ARGUMENTS ARE INPUTS.

NCOEFF = NUMBER OF FOURIER COEFFICIENTS FOR EACH STRESS DISTRIBUTION.
 NGE = NUMBER OF ACTIVE GAUSS ELEMENTS.
 THETA = ARRAY CONTAINING THE ANGULAR POSITION OF THE ELEMENTS AROUND THE CROSS SECTION.
 PHI = ARRAY CONTAINING THE ANGULAR ORIENTATION WITH RESPECT TO THE TANGENTIAL AXIS OF THE ELEMENTS.
 STIFFX = ARRAY CONTAINING THE READINGS OF THE ELEMENTS.
 XE = YOUNG'S MODULUS OF ELASTICITY.
 XYY = SHEAR MODULUS OF ELASTICITY.
 JJ = DILUTION'S RATIO.
 JJ = 3*NSCOEFF = TOTAL NUMBER OF COEFFICIENTS TO BE DETERMINED.
 NSCOEFF = SPECIFIED IN CALLING PROGRAM TO DETERMINE THE ESTIMATED STRESS DISTRIBUTION AND ALL CAN BE USED TO COMPARE THE ESTIMATED ONES FROM SUBPROGRAM CANTIL.

```

DIMENSION THETA(36), PHI(36), ALL(3,360,5), STRNEX(36), A(36),
      B(36), C(36), H(36), F(36), S(36), SL(360), SH(360), STA(10),
      STA(10), SLA(10), SLP(10), SHA(10), SHB(10),
      DIMENSION R(JJ), W(JJ)

```

DETERMINING COEFFICIENTS A, B AND C.

```

CALL CJMANA(XF, G, XNU, NGAGE, PHI, A, B, C, TTRANSV)
WRITE(6,98)
98 FORMAT(1H1)
      WRITE(5,33) ( A(N), N=1, NGAGE )
      WRITE(5,34) ( B(N), N=1, NGAGE )
      WRITE(6,35) ( C(N), N=1, NGAGE )
      WRITE(6,36) ( A(N), N=1, NGAGE ) READ ROW WISE // (15X, 6F15.5)
      WRITE(6,37) ( B(N), N=1, NGAGE ) READ ROW WISE // (15X, 6F15.5)
      WRITE(6,38) ( C(N), N=1, NGAGE ) READ ROW WISE // (15X, 6F15.5)
      WRITE(6,39) ( A(N), N=1, NGAGE ) READ ROW WISE // (15X, 6F15.5)
      WRITE(6,40) ( B(N), N=1, NGAGE ) READ ROW WISE // (15X, 6F15.5)
      WRITE(6,41) ( C(N), N=1, NGAGE ) READ ROW WISE // (15X, 6F15.5)

```

FOR A SET OF SIMULTANEOUS EQUATIONS TO SOLVE FOR NSMLX'S FOURIER COEFFICIENTS. IN MATRIX NOTATION: $(F)x(C)eff \cdot s = (h)$.

$\text{NSML} = \text{NSML} * 10^{16} / 2 + 1$
 $\text{NSMLX5} = \text{NSML} * 5$
 $\text{D12} = 3.14159265 / 180.0$

DETERMINE CONSTANTS H'S.

$\text{DO } 3 \quad L = 1, \text{ NSMLX6}$
 $L = L + 5$

$K = K + 1$
 $H(4) = 1 = \text{THETA}(1) * PI$
 $P1 = \text{COS}(AK * PI)$
 $C1 = \text{SIN}(AK * PI)$
 $H(L+1) = H(L+1) + STRNEX(1) * A(1) * C1$
 $H(L+2) = H(L+2) + STRNFX(1) * A(1) * S1$
 $H(L+3) = H(L+3) + STRNEX(1) * B(1) * C1$
 $H(L+4) = H(L+4) + STRNFX(1) * B(1) * S1$
 $H(L+5) = H(L+5) + STRNFX(1) * C(1) * C1$
4. CONTINUE

DETERMINE COEFFICIENTS F'S.

$\text{DO } 1 \quad L = 1, \text{ NSMLX6}$
 $F(L, M) = 0.0$
 $K = 1$
 $DO 2 \quad L = 1, \text{ NSMLX6}, 6$
 $AK = K$
 $K = K + 1$
 $LA = L$
 $L6 = L + 5$
 $DO 3 \quad J = 1, 5$
 $AJ = J + 1$
 $MA = M$
 $M6 = M + 5$
 $DO 4 \quad I = 1, 5$
 $AA = A(I) * A(I)$
 $AB = A(I) * B(I)$
 $AC = A(I) * C(I)$

GUAYR048
 GUAYR049
 GUAYR050
 GUAYR051
 GUAYR052
 GUAYR053
 GUAYR054
 GUAYR055
 GUAYR056
 GUAYR057
 GUAYR058
 GUAYR059
 GUAYR060
 GUAYR061
 GUAYR062
 GUAYR063
 GUAYR064
 GUAYR065
 GUAYR066
 GUAYR067
 GUAYR068
 GUAYR069
 GUAYR070
 GUAYR071
 GUAYR072
 GUAYR073
 GUAYR074
 GUAYR075
 GUAYR076
 GUAYR077
 GUAYR078
 GUAYR079
 GUAYR080
 GUAYR081
 GUAYR082
 GUAYR083
 GUAYR084
 GUAYR085
 GUAYR086
 GUAYR087
 GUAYR088
 GUAYR089
 GUAYR090
 GUAYR091
 GUAYR092
 GUAYR093
 GUAYR094
 GUAYR095

GUAYR098
 GUAYR099
 GUAYR100
 GUAYR101
 GUAYR102
 GUAYR103
 GUAYR104
 GUAYR105
 GUAYR106
 GUAYR107
 GUAYR108
 GUAYR109
 GUAYR110
 GUAYR111
 GUAYR112
 GUAYR113
 GUAYR114
 GUAYR115
 GUAYR116
 GUAYR117
 GUAYR118
 GUAYR119
 GUAYR120
 GUAYR121
 GUAYR122
 GUAYR123
 GUAYR124
 GUAYR125
 GUAYR126
 GUAYR127
 GUAYR128
 GUAYR129
 GUAYR130
 GUAYR131
 GUAYR132
 GUAYR133
 GUAYR134
 GUAYR135
 GUAYR136
 GUAYR137
 GUAYR138
 GUAYR139
 GUAYR140
 GUAYR141
 GUAYR142
 COR-
 INATE ROWS AND COLUMNS OF ALL-ZERO ELEMENTS IN MATRIX F,
 CONTINUE

2

1

KEEPING THE FOURIER COEFFICIENTS OF THE TERMS CONTAINING COR-
 SSES DURING ELEMENTS IN CONSTANT MATRIX H. THE MATRIX EQUATION IS
 CONVERTED TO $(R) \times (COEFFS) = (w)$.

```

W(1,1) = H(1,1)
R(1,1) = F(1,1)
W(2,1) = H(3)
R(1,2) = F(1,3)
W(3,1) = H(5)
R(1,3) = F(1,5)
F(1,1) = F(1,1)
F(2,1) = F(3,1)
F(2,2) = F(3,3)
F(2,3) = F(3,5)
R(3,1) = F(5,1)
R(3,2) = F(5,3)
R(3,3) = F(5,5)
DO 50 M = 4, JJ
W(M) = H(M+3)
R(1,M) = F(1,M+3)
F(2,M) = F(3,M+3)
R(3,M) = F(5,M+3)
DO 50 L = 1, 3
      R(M,L) = R(L,M)
      DO 51 L = 4, JJ
      DO 51 M = 4, JJ
      51 R(L,M) = F(L+3,M+3)
  
```

C SOLVE THE SET OF SIMULTANEOUS EQUATIONS TO DETERMINE JJ FOURIER
C COEFFICIENTS.

```

CALL DSIM3(R,w,JJ,KS)
WRITE(6,38)
38  WRITE(6,12) NCOEFF
      FORMAT(7(/,15X,'COEFFICIENTS'))
      WRITE(6,10C) KS
      FORMAT(7(/,15X,'KS = ''I2''))
      100 1 0F SOLUTION: 0 FOR NORMAL, 1 FOR SINGULAR, J
      120 1 0F
  
```

C GROUP THE FOURIER COEFFICIENTS FOR THE TANGENTIAL STRESS IN THE
C ARRAY STA, CORRESPONDING TO THE TERM CONTAINING COS(N THETA), AND IN
C THE ARRAY STB CORRESPONDINGLY FOR THE TERMS IN SIN(N THETA), FOR
C N = 1, 2, (NCOEFF/2 + 1).
C SAME FOR LONGITUDINAL STRESS WITH SLA AND SLB, AND FOR SHEAR STRESS


```

50=1   FORMAT(//2G,42(2X,5'CJS',12'(THETA)',4F14.5/15X,'LUNG.',4F14.5')//)
      15X,TANG.,4F14.5/15X,12'(THETA)',4F14.5')//
      GU TO 600
      M = 1
      KS = 1
      I = 2
      LA = 4
      WRITE(6,7001) (K,K,KS=M,KS)=(STA(N),STB(N),N=1,LA),
      1  FORM(//2GX,3(2X,5'CJS',12'(THETA)',4F14.5/15X,'LUNG.',4F14.5')//
      15X,TANG.,6E14.5/15X,12'(THETA)',4F14.5/15X,'SHEAR.',6E14.5')//
      GU TO 600
      M = 1
      KS = 3
      I = 2
      LA = 4
      WRITE(6,7001) (K,K,KS=M,KS)=(STA(N),STB(N),N=1,LA),
      1  (SLA(N),SLB(N),N=1,LA), (SHA(N),SHB(N),N=1,LA),
      M = 4
      KS = 4
      I = 5
      LA = 5
      WRITE(6,3001) (K,K,KS=M,KS)=(STA(N),STB(N),N=1,LA),
      1  (SLA(N),SLB(N),N=1,LA), (SHA(N),SHB(N),N=1,LA),
      M = 4
      KS = 4
      I = 2
      LA = 2
      WRITE(6,7001) (K,K,KS=M,KS)=(STA(N),STB(N),N=1,LA),
      1  GU TO 600
      M = 1
      KS = 2
      I = 2
      LA = 4
      WRITE(6,7001) (K,K,KS=M,KS)=(STA(N),STB(N),N=1,LA),
      1  (SLA(N),SLB(N),N=1,LA), (SHA(N),SHB(N),N=1,LA),
      M = 4
      KS = 5
      I = 5
      LA = 6
      WRITE(6,5001) (K,K,KS=M,KS)=(STA(N),STB(N),N=1,LA),
      1  CONTINUE
      600
C COMPUTE RESULTANT STRESS COMPONENTS EVERY 10 DEGREES AROUND THE
C CROSS SECTION.
C
      00 5   I = 1, 36
      ST(I) = 0.0
      SL(I) = 0.0
      SH(I) = 0.0

```

```

C          PI3 = 1.0 * PI2
C          DC6 = 1.0 * 1, 36
C          AI = 1.0 * PI3
C          PI = AI * PI3
C          AN = N - 1, NSML
C          AN = COS(AN*PI)
C          SI = SIN(AN*PI)
C          ST(I) = SI(I) + STA(N)*CI + ST3(N)*SI
C          SL(I) = SL(I) + SLA(N)*CI + SLB(N)*SI
C          SH(I) = SH(I) + SHA(N)*CI + SHB(N)*SI
C
C          PREPARE PRINT-OUT OF THE COMPUTED STRESSES AND THE CORRESPONDING
C          HORIZONTALINES CONTAINED IN ALL, IDENTIFIED BY THE NUMBER OF
C          THE CROSS SECTION NSECT.
C
C          K = 1
C          WRITE(6, 38)
C          DO 7 N = 1, 36
C          L = 10*(N - 1)
C          J = L + 1
C          IF( K .LT. 13 ) GO TO 956
C
C          K = 1
C          WRITE(6, 88)
C          956  WRITE(6, 11) L, J, (ALL(I, J, NSECT), I = 1, 3), ST(N), SL(N), SH(N)
C          11  FORMAT(1/15X, !THTA = !14, !DEGREES, !5X, !POINT NO; !14, !
C          1 15X, !STRESSES, !9X, !TANG, !12X, !LONG, !12X, !SHEAR; !14, !
C          2 15X, !ACTUAL, !3E17.5/15X, !COMPUTED, !3E17.5)
C          7  K = -K + 1
C          RETURN
C          END

```

```

SUBROUTINE CUMANA( E, G, V, NPOINT, P, A, R, C, AK )
C TO COMPUTE COEFFICIENTS A, B AND C TO BE USED IN PROGRAM GUAYRA.
C
C DIMENSION F(36), A(36), B(36), C(36)
C
P12 = 3.14159265/180.0
C0_4 = 1.0
N = P(N)*P12
P1 = P(N)*P12
A(N) = ((1.0-AK)*((1.0+V)*SIN(P1)**2)/(E*(1.0-AK**2)))
B(N) = ((1.0-C-AK)*((1.0+V)*COS(P1)**2)/(E*(1.0-AK**2)))
C(N) = ((1.0-C-AK)*SIN(2.0*p1))/(2.0*C*(1.0-AK**2))
4   RETURN
END

```

4

SUBROUTINE TUNATE(E,G,V,K,AL,NBIG,T,P,S,AK)

TO COMPUTE THE THEORETICAL READINGS OF A SET OF NBIG STRAIN GAGE ELEMENTS WITH TRANSVERSE SENSITIVITY AK.

ANGULAR POSITIONS OF THE ELEMENTS ARE CONTAINED IN THE ARRAY OF ANGULAR ORIENTATION OF THE ELEMENTS WITH RESPECT TO THE TANGENTIAL AXIS ARE CONTAINED IN THE ARRAY OF ANGLES P. ELEMENTS OF THE STRESSES AT EACH POSITION ARE CONTAINED IN THE COLUMNS OF THE MATRIX OF THE ELEMENTS OF THE CROSS SECTION T) IDENTIFY THE ELEMENT STRESSES.

CURRENT YOUNG'S MODULUS OF ELASTICITY.

THIS SHEAR MODULUS OF ELASTICITY.

THIS POISSON'S RATIO.

THIS THE OUTPUT SET OF READINGS.

```
DIMENSION AL(3,36C,5),T(36), P(36), S(36)
PI2 = 3.14159265/180.
DO 4 N = 1, NBIG
   J = T(N) + 1
   P1 = P(N)*PI2
   S1 = AL(1,J,K)
   S2 = AL(2,J,K)
   S3 = AL(3,J,K)
   A = (((1.0-AK**V)-(1.0-AK)*(1.0+V)*SIN(P1)**2)/(E*(1.0-AK**2)))
   B = (((1.0-AK**V)-(1.0-AK)*(1.0+V)*COS(P1)**2)/(F*(1.0-AK**2)))
   C = (((1.0-AK)*SIN(2.0*P1))/(2.0*G*(1.0-AK**2)))
   S(N) = A*S1 + B*S2 + C*S3
   COUNTNE
END
```

SUBROUTINE PLANCA(CO,BF,N,ALT)
INTRODUCES RANDOM ERRORS IN THE ELEMENTS OF ONE DIMENSIONAL ARRAY.
CO IS THE ARRAY INPUT AND BF IS THE OUTPUT.
N IS THE NUMBER OF ELEMENTS.
ALT IS A MULTIPLIER TO PRODUCE PLUS OR MINUS (ALT)*(10%) MAXIMUM
ERROR.

DIMENSION CO(36), BE(36)
 RH0 =
 CO 5 J = 1, N
 G = J RHC + 3.6227
 IF (RHC - 5.995) 12, 11, 11
 RHO = RHC - 8.4153
 XX = RH0***(G/16.6 + 3.1416)
 XX = XX
 AX = XX - AX
 BX = XX - BX
 CX = 1G.J * BX
 TT = RF(J)
 PERC = 01*ALT
 GQ = PEKC*CKX#1
 KKX = CKX
 DKKX = KKKX
 FKKX = DKKX/2.0
 LKKX = FKKX
 GKKX = LKKX - GKKX
 CKX = FKX - C(5)^(2,1,2)
 IFC(5) = TT + GQ
 CO(J) = TT - GQ
 CONTINUE
 PERC = PERC*100.C.O
 WRITE(6,60) PERC
 60 FORMAT(1H,15X,'ALTERATION OF THE STRAIN GAGE READINGS! //'
 1 15X,'MAXIMLN ERROR INTRODUCED!', F6.1, 2, // 34X, 'ORIGINAL',
 2 17X,'ALTERED.'
 WRITE(6,63) (BE(J), CO(J), J = 1, N)
 FORWAT(26X, 2E15.5)
 RETURN
 END

MODULUS OF ELASTICITY C 300000E 08PSI
 SHEAR MODULUS OF ELASTICITY 0.11538E 08 PSI
 POISSON RATIO 0.3000E 00

NUMBER OF GAGES 36
 TRANSVERSE SENSITIVITY FACTOR OF THE GAGES = 0 %
 NUMBER OF GAGES FUNCTIONING PROPERLY 36
 GAGES FUNCTIONING IMPROPERLY ... NONE

ANGULAR POSITION OF THE GAGE IN THE CROSS SECTION, READ ROW WISE

0.0	C.0	0.0	0.30000E 02	0.30000E 02	0.30000E 02
0.60000E 02	C.60000E 02	0.60000E 02	0.90000E 02	0.90000E 02	0.90000E 02
0.12000E 03	C.12000E 03	0.12000E 03	0.15000E 03	0.15000E 03	0.15000E 03
0.18000E 03	C.18000E 03	0.18000E 03	0.21000E 03	0.21000E 03	0.21000E 03
0.24000E 03	C.24000E 03	0.24000E 03	0.27000E 03	0.27000E 03	0.27000E 03
0.30000E 03	C.30000E 03	0.30000E 03	0.33000E 03	0.33000E 03	0.33000E 03

ANGULAR ORIENTATION OF THE GAGES, READ ROW WISE

0.0	C.60000E 02	0.12000E 03	0.0	0.60000E 02	0.60000E 02
0.0	C.60000E 02	0.12000E 03	0.0	0.60000E 02	0.60000E 02
0.0	C.60000E 02	0.12000E 03	0.0	0.60000E 02	0.60000E 02
0.0	C.60000E 02	0.12000E 03	0.0	0.60000E 02	0.60000E 02
0.0	C.60000E 02	0.12000E 03	0.0	0.60000E 02	0.60000E 02

READINGS OF THE GAGE ELEMENTS, READ ROW WISE.

0.15134E-04	C.64178E-06	0.19479E-06	0.15127E-04	0.52988E-06	0.33917E-06
0.15118E-04	C.40066E-06	0.50589E-06	0.15111E-04	0.28876E-06	0.65028E-06
0.15107E-04	C.22415E-06	0.73364E-06	0.15107E-04	0.22415E-06	0.73364E-06
0.15111E-04	C.28876E-06	0.65028E-06	0.15118E-04	0.40066E-06	0.50589E-06
0.15127E-04	C.52988E-06	0.33917E-06	0.15134E-04	0.64178E-06	0.19479E-06
0.15138E-04	C.70639E-06	0.11143E-06	0.15138E-04	0.70639E-06	0.11143E-06

COEFFICIENTS A, READ ROW WISE

0.	333333E-07	C. 8333336E-09	0. 833333E-09	0. 333333E-07	0. 8333336E-07	0. 833333E-09
0.	333333E-07	C. 8333336E-09	0. 833333E-09	0. 333333E-07	0. 8333336E-07	0. 833333E-09
0.	333333E-07	C. 8333336E-09	0. 833333E-09	0. 333333E-07	0. 8333336E-07	0. 833333E-09
0.	333333E-07	C. 8333336E-09	0. 833333E-09	0. 333333E-07	0. 8333336E-07	0. 833333E-09
0.	333333E-07	C. 8333336E-09	0. 833333E-09	0. 333333E-07	0. 8333336E-07	0. 833333E-09
0.	333333E-07	C. 8333336E-09	0. 833333E-09	0. 333333E-07	0. 8333336E-07	0. 833333E-09
0.	333333E-07	C. 8333336E-09	0. 833333E-09	0. 333333E-07	0. 8333336E-07	0. 833333E-09

COEFFICIENTS B, READ ROW WISE

-C.	10000E-07	C. 22500E-07	0. 22500E-07	-0. 10000E-07	0. 22500E-07	0. 22500E-07
-0.	10000E-07	C. 22500E-07	0. 22500E-07	-0. 10000E-07	0. 22500E-07	0. 22500E-07
-C.	10000E-07	C. 22500E-07	0. 22500E-07	-0. 10000E-07	0. 22500E-07	0. 22500E-07
-0.	10000E-07	C. 22500E-07	0. 22500E-07	-0. 10000E-07	0. 22500E-07	0. 22500E-07
-0.	10000E-07	C. 22500E-07	0. 22500E-07	-0. 10000E-07	0. 22500E-07	0. 22500E-07
-0.	10000E-07	C. 22500E-07	0. 22500E-07	-0. 10000E-07	0. 22500E-07	0. 22500E-07

COEFFICIENTS C, READ ROW WISE

-0.	0.	C. 37528E-07	-0. 37528E-07	-0. 0.	0. 37528E-07	-0. 37528E-07
-0.	0.	C. 37528E-07	-0. 37528E-07	-0. 0.	0. 37528E-07	-0. 37528E-07
-0.	0.	C. 37528E-07	-0. 37528E-07	-0. 0.	0. 37528E-07	-0. 37528E-07
-0.	0.	C. 37528E-07	-0. 37528E-07	-0. 0.	0. 37528E-07	-0. 37528E-07
-0.	0.	C. 37528E-07	-0. 37528E-07	-0. 0.	0. 37528E-07	-0. 37528E-07
-0.	0.	C. 37528E-07	-0. 37528E-07	-0. 0.	0. 37528E-07	-0. 37528E-07

EXPANSION WITH 11 FOURIER COEFFICIENTS

KS = 0 KS IS AN INDEX INDICATING TYPE OF SOLUTION: 0 FOR NORMAL, 1 FOR SINGULAR

FOURIER COEFFICIENTS

CONSTANTS	C3
TANG.	0.45455E
LONG.	0.28943E
SHEAR	0.56926E

	COS 1(THETA)	SIN 1(THETA)	COS 2(THETA)	SIN 2(THETA)	COS 3(THETA)	SIN 3(THETA)
TANG.	-0.51868E-03	-0.95258E-04	0.16030E-04	-0.62048E-04	0.94102E-05	-0.21859E-04
LONG.	-0.11386E-01	0.11385E-01	-0.83788E-04	-0.39759E-04	-0.68131E-04	-0.23244E-04
SHEAR	0.53861E-01	-0.53860E-01	-0.82367E-05	0.46219E-05	-0.20428E-05	-0.13867E-05

Note: If the system of equations for the Fourier coefficients is singular, this fact is signaled by KS = 1. Any output which follows is invalid.

	COS 4(THETA)	SIN 4(THETA)	COS 5(THETA)	SIN 5(THETA)
TANG.	0.38785E-04	-0.16227E-05	-0.13316E-05	0.42612E-04
LONG.	-0.59643E-04	-0.35988E-04	0.16735E-06	0.61955E-04
SHEAR	0.56175E-06	-0.41939E-06	0.42750E-06	-0.26063E-06

THETA =	0 DEGREES	POINT NO.	1	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E C3	0.17552E 01	0.59553E 01	
COMPUTED	0.45455E C3	0.17554E 01	0.59553E 01	
THETA =	10 DEGREES	POINT NO.	11	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E C3	0.19702E 01	0.49382E 01	
COMPUTED	0.45455E C3	0.19704E 01	0.49382E 01	
THETA =	20 DEGREES	POINT NO.	21	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E C3	0.22133E 01	0.37884E 01	
COMPUTED	0.45455E C3	0.22135E 01	0.37884E 01	
THETA =	30 DEGREES	POINT NO.	31	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E C3	0.24770E 01	0.25407E 01	
COMPUTED	0.45454E C3	0.24773E 01	0.25407E 01	
THETA =	40 DEGREES	POINT NO.	41	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E C3	0.27534E 01	0.12331E 01	
COMPUTED	0.45454E C3	0.27538E 01	0.12331E 01	
THETA =	50 DEGREES	POINT NO.	51	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E C3	0.30341E 01	-0.94609E-01	
COMPUTED	0.45454E C3	0.30345E 01	-0.94599E-01	
THETA =	60 DEGREES	POINT NO.	61	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E C3	0.33104E 01	-0.14022E 01	
COMPUTED	0.45455E C3	0.33110E 01	-0.14022E 01	
THETA =	70 DEGREES	POINT NO.	71	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E C3	0.35742E 01	-0.26498E 01	
COMPUTED	0.45455E C3	0.35748E 01	-0.26498E 01	
THETA =	80 DEGREES	POINT NO.	81	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E C3	0.38172E 01	-0.37997E 01	
COMPUTED	0.45455E C3	0.38178E 01	-0.37997E 01	
THETA =	90 DEGREES	POINT NO.	91	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E C3	0.40322E 01	-0.48168E 01	
COMPUTED	0.45455E C3	0.40328E 01	-0.48168E 01	
THETA =	100 DEGREES	POINT NO.	101	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E C3	0.42126E 01	-0.56702E 01	
COMPUTED	0.45455E C3	0.42132E 01	-0.56702E 01	
THETA =	110 DEGREES	POINT NO.	111	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E C3	0.43530E 01	-0.63341E 01	
COMPUTED	0.45455E C3	0.43535E 01	-0.63341E 01	

THETA = 120 DEGREES	POINT NO.	121	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.44490E 01	-0.67882E 01
COMPUTED	0.45455E 03	0.44495E 01	-0.67882E 01
THETA = 130 DEGREES	POINT NO.	131	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.44977E 01	-0.70188E 01
COMPUTED	0.45455E 03	0.44982E 01	-0.70188E 01
THETA = 140 DEGREES	POINT NO.	141	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.44977E 01	-0.70188E 01
COMPUTED	0.45455E 03	0.44983E 01	-0.70188E 01
THETA = 150 DEGREES	POINT NO.	151	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.44490E 01	-0.67882E 01
COMPUTED	0.45455E 03	0.44496E 01	-0.67882E 01
THETA = 160 DEGREES	POINT NO.	161	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.43530E 01	-0.63341E 01
COMPUTED	0.45455E 03	0.43537E 01	-0.63341E 01
THETA = 170 DEGREES	POINT NO.	171	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.42126E 01	-0.56703E 01
COMPUTED	0.45455E 03	0.42133E 01	-0.56702E 01
THETA = 180 DEGREES	POINT NO.	181	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.40322E 01	-0.48168E 01
COMPUTED	0.45455E 03	0.40328E 01	-0.48168E 01
THETA = 190 DEGREES	POINT NO.	191	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.38172E 01	-0.37997E 01
COMPUTED	0.45455E 03	0.38178E 01	-0.37997E 01
THETA = 200 DEGREES	POINT NO.	201	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.35742E 01	-0.26498E 01
COMPUTED	0.45455E 03	0.35747E 01	-0.26498E 01
THETA = 210 DEGREES	POINT NO.	211	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.33105E 01	-0.14022E 01
COMPUTED	0.45455E 03	0.33110E 01	-0.14022E 01
THETA = 220 DEGREES	POINT NO.	221	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.30341E 01	-0.94623E-01
COMPUTED	0.45455E 03	0.30347E 01	-0.94618E-01
THETA = 230 DEGREES	POINT NO.	231	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.27534E 01	0.12331E 01
COMPUTED	0.45455E 03	0.27541E 01	0.12331E 01

THETA = 240 DEGREES	POINT NO.	241	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E C3	0.24770E 01	0.25407E 01
COMPUTED	0.45455E C3	0.24777E 01	0.25407E 01
THETA = 250 DEGREES	POINT NO.	251	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E C3	0.22133E 01	0.37883E 01
COMPUTED	0.45455F C3	0.22139E 01	0.37883E 01
THETA = 260 DEGREES	POINT NO.	261	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E C3	0.19702E 01	0.49382E 01
COMPUTED	0.45455E C3	0.19708E 01	0.49382E 01
THETA = 270 DEGREES	POINT NO.	271	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E C3	0.17552E 01	0.59553E 01
COMPUTED	0.45455E C3	0.17558E 01	0.59553E 01
THETA = 280 DEGREES	POINT NO.	281	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E C3	0.15748E 01	0.68088F 01
COMPUTED	0.45455E C3	0.15754E 01	0.68087E 01
THETA = 290 DEGREES	POINT NO.	291	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E C3	0.14345E 01	0.74726E 01
COMPUTED	0.45455E C3	0.14351E 01	0.74726E 01
THETA = 300 DEGREES	POINT NO.	301	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E C3	0.13385E 01	0.79267E 01
COMPUTED	0.45455E C3	0.13392E 01	0.79267E 01
THETA = 310 DEGREES	POINT NO.	311	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E C3	0.12898E 01	0.81573F 01
COMPUTED	0.45455E C3	0.12905E 01	0.81573E 01
THETA = 320 DEGREES	POINT NO.	321	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E C3	0.12897E 01	0.81573F 01
COMPUTED	0.45455E C3	0.12904E 01	0.81573E 01
THETA = 330 DEGREES	POINT NO.	331	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E C3	0.13385E 01	0.79267E 01
COMPUTED	0.45455E C3	0.13390E 01	0.79267E 01
THETA = 340 DEGREES	POINT NO.	341	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E C3	0.14345E 01	0.74726E 01
COMPUTED	0.45455E C3	0.14348E 01	0.74726E 01
THETA = 350 DEGREES	POINT NO.	351	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E C3	0.15748E 01	0.68088E 01
COMPUTED	0.45455E C3	0.15751E 01	0.68088E 01

EXPANSION WITH 3 FOURIER COEFFICIENTS

KS = 0 KS IS AN INDEX INDICATING TYPE OF SOLUTION: 0 FOR NORMAL, 1 FOR SINGULAR

FOURIER COEFFICIENTS

CONSTANTS

TANG.	0.45455E	03
LONG.	0.28943E	01
SHEAR	0.56926E	00

COS 1(THETA)

-0.51868E-03	SIN 1(THETA)
-0.11386E 01	-0.95253E-04
0.53861E 01	0.11385E 01

TANG.

LONG.

SHEAR

THETA =	0 DEGREES	POINT NO.	1	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E 03	0.17552E 01		0.59553E 01
COMPUTED	0.45455F 03	0.17556E 01		0.59553E 01
THETA =	10 DEGREES	POINT NO.	11	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455F 03	0.19702E 01		0.49382E 01
COMPUTED	0.45455E 03	0.19706E 01		0.49382E 01
THETA =	20 DEGREES	POINT NO.	21	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E 03	0.22133E 01		0.37884E 01
COMPUTED	0.45455E 03	0.22137F 01		0.37884F 01
THETA =	30 DEGREES	POINT NO.	31	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E 03	0.24770E 01		0.25407E 01
COMPUTED	0.45455F 03	0.24774E 01		0.25407E 01
THETA =	40 DEGREES	POINT NO.	41	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E 03	0.27534F 01		0.12331E 01
COMPUTED	0.45455F 03	0.27538E 01		0.12331E 01
THETA =	50 DEGREES	POINT NO.	51	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455F 03	0.30341E 01		-0.94609E-01
COMPUTED	0.45455F 03	0.30345E 01		-0.94604E-01
THETA =	60 DEGREES	POINT NO.	61	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455F 03	0.33104E 01		-0.14022E 01
COMPUTED	0.45455F 03	0.33109E 01		-0.14022E 01
THETA =	70 DEGREES	POINT NO.	71	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E 03	0.35742E 01		-0.26498E 01
COMPUTED	0.45455F 03	0.35746E 01		-0.26498E 01
THETA =	80 DEGREES	POINT NO.	81	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E 03	0.38172E 01		-0.37997E 01
COMPUTED	0.45455F 03	0.38177E 01		-0.37997E 01
THETA =	90 DEGREES	POINT NO.	91	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E 03	0.40322F 01		-0.48168E 01
COMPUTED	0.45455F 03	0.40327E 01		-0.48168E 01
THETA =	100 DEGREES	POINT NO.	101	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E 03	0.42126E 01		-0.56702E 01
COMPUTED	0.45455F 03	0.42132E 01		-0.56702E 01
THETA =	110 DEGREES	POINT NO.	111	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455F 03	0.43530E 01		-0.63341E 01
COMPUTED	0.45455F 03	0.43535E 01		-0.63341F 01

THETA = 120 DEGREES	POINT NO.	121	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.44490E 01	-0.67882E 01
COMPUTED	0.45455E 03	0.44495E 01	-0.67882E 01
THETA = 130 DEGREES	POINT NO.	131	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.44977E 01	-0.70188E 01
COMPUTED	0.45455E 03	0.44983E 01	-0.70188E 01
THETA = 140 DEGREES	POINT NO.	141	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.44977E 01	-0.70188E 01
COMPUTED	0.45455E 03	0.44983E 01	-0.70188E 01
THETA = 150 DEGREES	POINT NO.	151	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.44490E 01	-0.67882E 01
COMPUTED	0.45455E 03	0.44496E 01	-0.67882E 01
THETA = 160 DEGREES	POINT NO.	161	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.43530E 01	-0.63341E 01
COMPUTED	0.45455E 03	0.43536E 01	-0.63341E 01
THETA = 170 DEGREES	POINT NO.	171	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.42126E 01	-0.56703E 01
COMPUTED	0.45455E 03	0.42133E 01	-0.56702E 01
THETA = 180 DEGREES	POINT NO.	181	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.40322E 01	-0.48168E 01
COMPUTED	0.45455E 03	0.40329E 01	-0.48168E 01
THETA = 190 DEGREES	POINT NO.	191	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.38172E 01	-0.37997E 01
COMPUTED	0.45455E 03	0.38179E 01	-0.37997E 01
THETA = 200 DEGREES	POINT NO.	201	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.35742E 01	-0.26498E 01
COMPUTED	0.45455E 03	0.35749E 01	-0.26498E 01
THETA = 210 DEGREES	POINT NO.	211	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.33105E 01	-0.14022E 01
COMPUTED	0.45455E 03	0.33111E 01	-0.14022E 01
THETA = 220 DEGREES	POINT NO.	221	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.30341E 01	-0.94623E-01
COMPUTED	0.45455E 03	0.30347E 01	-0.94625E-01
THETA = 230 DEGREES	POINT NO.	231	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.27534E 01	0.12331E 01
COMPUTED	0.45455E 03	0.27541E 01	0.12331E 01

THETA = 240 DEGREES	POINT NO.	241	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.24770E 01	0.25407E 01
COMPUTED	0.45455E 03	0.24777E 01	0.25407E 01
THETA = 250 DEGREES	POINT NO.	251	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.22133E 01	0.37883E 01
COMPUTED	0.45455E 03	0.22139E 01	0.37883E 01
THETA = 260 DEGREES	POINT NO.	261	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.19702E 01	0.49382E 01
COMPUTED	0.45455E 03	0.19708E 01	0.49382E 01
THETA = 270 DEGREES	POINT NO.	271	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.17552E 01	0.59553E 01
COMPUTED	0.45455E 03	0.17558E 01	0.59553E 01
THETA = 280 DEGREES	POINT NO.	281	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.15748E 01	0.68088E 01
COMPUTED	0.45455E 03	0.15754E 01	0.68087E 01
THETA = 290 DEGREES	POINT NO.	291	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.14345E 01	0.74726E 01
COMPUTED	0.45455E 03	0.14350E 01	0.74726E 01
THETA = 300 DEGREES	POINT NO.	301	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.13385E 01	0.79267E 01
COMPUTED	0.45455E 03	0.13390E 01	0.79267E 01
THETA = 310 DEGREES	POINT NO.	311	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.12898E 01	0.81573E 01
COMPUTED	0.45455E 03	0.12903E 01	0.81573E 01
THETA = 320 DEGREES	POINT NO.	321	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.12897E 01	0.81573E 01
COMPUTED	0.45455E 03	0.12902E 01	0.81573E 01
THETA = 330 DEGREES	POINT NO.	331	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.13385E 01	0.79267E 01
COMPUTED	0.45455E 03	0.13390E 01	0.79267E 01
THETA = 340 DEGREES	POINT NO.	341	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.14345E 01	0.74726E 01
COMPUTED	0.45455E 03	0.14349E 01	0.74726E 01
THETA = 350 DEGREES	POINT NO.	351	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.15748E 01	0.68088E 01
COMPUTED	0.45455E 03	0.15752E 01	0.68088E 01

MODULUS OF ELASTICITY C 300000E 08PSI
 SHEAR MODULUS OF ELASTICITY 0.11538E 08 PSI
 POISSON RATIO 0.3000E 00

NUMBER OF GAGES 36
 TRANSVERSE SENSITIVITY FACTOR OF THE GAGES = 0 %
 NUMBER OF GAGES FUNCTIONING PROPERLY 30
 GAGES FUNCTIONING IMPROPERLY 2, 12, 22, 24, 25, 33,

ANGULAR POSITION OF THE GAGE IN THE CROSS SECTION, READ ROW WISE

0.0	C.0	0.30000E 02				
0.60000E 02	C.60000E 02	0.90000E 02				
0.12000E 03	C.15000E 03	0.15000E 03	0.15000E 03	0.15000E 03	0.18000E 03	0.18000E 03
0.18000E 03	C.21000E 03	0.24000E 03	0.24000E 03	0.24000E 03	0.27000E 03	0.27000E 03
0.27000E 03	C.30000E 03	0.30000E 03	0.33000E 03	0.33000E 03	0.33000E 03	0.33000E 03

ANGULAR ORIENTATION OF THE GAGES, READ ROW WISE

0.0	C.12000E 03	0.0	0.60000E 02	0.60000E 02	0.12000E 03	0.60000E 02
0.60000E 02	C.12000E 03	0.0	0.60000E 02	0.60000E 02	0.0	0.60000E 02
0.12000E 03	C.0	0.60000E 02	0.60000E 02	0.12000E 03	0.0	0.60000E 02
0.12000E 03	C.60000E 02	0.0	0.60000E 02	0.0	0.60000E 02	0.60000E 02
0.12000E 03	C.0	0.60000E 02	0.0	0.0	0.60000E 02	0.12000E 03

READINGS OF THE GAGE ELEMENTS, READ ROW WISE.

0.15134E-04	C.19479E-C6	0.15127E-04	0.52988E-C6	0.33917E-06	0.15118E-04
0.40066E-06	C.50589E-06	0.15111E-04	0.28876E-06	0.15107E-04	0.22415E-06
0.73364E-06	C.15107E-04	0.22415E-06	0.73364E-06	0.15111E-04	0.28876E-06
0.65028E-06	C.40066E-06	0.52988E-06	0.33917E-06	0.15134E-04	0.64178E-06
0.19479E-06	C.15138E-04	0.70639E-06	0.15138E-04	0.70639E-06	0.11143F-06

COEFFICIENTS A, READ ROW WISE

0.333333E-07	C.833333E-09	0.333333E-07	0.83336E-09	0.833333E-09	0.333333E-07
0.833333E-09	C.833333E-09	0.333333E-07	0.83336E-09	0.333333E-07	0.833333E-09
0.833333E-09	C.333333E-07	0.83336E-09	0.833333E-09	0.333333E-07	0.833333E-09
0.833333E-09	C.83336E-C9	0.83336E-09	0.833333E-09	0.333333E-07	0.833333E-09
0.833333E-09	C.333333E-07	0.83336E-09	0.333333E-07	0.833333E-07	0.833333E-09

COEFFICIENTS B, READ ROW WISE

-0.10000E-07	C.22500E-07	-0.10000E-07	0.22500E-07	0.22500E-07	-0.10000E-07
0.22500E-07	C.22500E-07	-0.10000E-07	0.22500E-07	0.22500E-07	-0.10000E-07
0.22500E-07	-C.10000E-07	0.22500E-07	0.22500E-07	0.22500E-07	0.22500E-07
0.22500E-07	C.22500E-07	0.22500E-07	0.22500E-07	0.22500E-07	0.22500E-07
0.22500E-07	-C.10000E-07	0.22500E-07	-0.10000E-07	0.22500E-07	-0.10000E-07

COEFFICIENTS C, READ ROW WISE

-0.0	-C.37528E-07	-0.0	0.37528E-07	0.37528E-07	-0.0
0.37528E-07	-C.37528E-07	-0.0	0.37528E-07	0.37528E-07	-0.0
-0.37528E-07	-C.0	0.37528E-07	-0.37528E-07	-0.37528E-07	0.37528E-07
-0.37528E-07	C.37528E-07	0.37528E-07	-0.37528E-07	0.37528E-07	-0.37528E-07
-0.37528E-07	-C.0	0.37528E-07	-0.0	0.37528E-07	-0.0

EXPANSION WITH 3 FOURIER COEFFICIENTS

K_S = 0 K_S IS AN INDEX INDICATING TYPE OF SOLUTION: 0 FOR NORMAL, 1 FOR SINGULAR

FOURIER COEFFICIENTS

CONSTANTS

TANG.	0.45455E	03
LONG.	0.28943E	01
SHEAR	0.56921E	00

COS 1(THETA)

TANG.	-0.94754E	-03
LONG.	-0.11387E	01
SHEAR	0.53861E	01

SIN 1(THETA)

TANG.	-0.57504E	-03
LONG.	0.11384E	01
SHEAR	-0.53860E	01

THETA =	0 DEGREES	POINT NO.	1	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E 03	0.17552E 01	0.59553E 01	
COMPUTED	0.45454E 03	0.17556E 01	0.59553E 01	
THETA =	10 DEGREES	POINT NO.	11	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E 03	0.19702E 01	0.49382E 01	
COMPUTED	0.45454E 03	0.19706E 01	0.49382E 01	
THETA =	20 DEGREES	POINT NO.	21	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E 03	0.22133E 01	0.37884E 01	
COMPUTED	0.45454E 03	0.22136E 01	0.37884E 01	
THETA =	30 DEGREES	POINT NO.	31	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E 03	0.24770E 01	0.25407E 01	
COMPUTED	0.45454E 03	0.24774E 01	0.25407E 01	
THETA =	40 DEGREES	POINT NO.	41	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E 03	0.27534E 01	0.12331E 01	
COMPUTED	0.45454E 03	0.27538E 01	0.12331E 01	
THETA =	50 DEGREES	POINT NO.	51	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E 03	0.30341E 01	-0.94609E-01	
COMPUTED	0.45454E 03	0.30344E 01	-0.94596E-01	
THETA =	60 DEGREES	POINT NO.	61	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E 03	0.33104E 01	-0.14022E 01	
COMPUTED	0.45454E 03	0.33108E 01	-0.14022E 01	
THETA =	70 DEGREES	POINT NO.	71	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E 03	0.35742E 01	-0.26498E 01	
COMPUTED	0.45454E 03	0.35746E 01	-0.26498E 01	
THETA =	80 DEGREES	POINT NO.	81	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E 03	0.38172E 01	-0.37997E 01	
COMPUTED	0.45454E 03	0.38177E 01	-0.37997E 01	
THETA =	90 DEGREES	POINT NO.	91	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E 03	0.40322E 01	-0.48168E 01	
COMPUTED	0.45454E 03	0.40327E 01	-0.48168E 01	
THETA =	100 DEGREES	POINT NO.	101	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E 03	0.42126E 01	-0.56702E 01	
COMPUTED	0.45454E 03	0.42131E 01	-0.56703E 01	
THETA =	110 DEGREES	POINT NO.	111	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E 03	0.43530E 01	-0.63341E 01	
COMPUTED	0.45455E 03	0.43535E 01	-0.63341E 01	

MODULUS OF ELASTICITY C.30000E 08PSI
 SHEAR MODULUS OF ELASTICITY 0.11538E 08 PSI
 POISSON RATIO 0.3000E 0C

NUMBER OF GAGES 36
 TRANSVERSE SENSITIVITY FACTOR OF THE GAGES = 0 %
 NUMBER OF GAGES FUNCTIONING PROPERLY 36
 GAGES FUNCTIONING IMPROPERLY ... NONE

ANGULAR POSITION OF THE GAGE IN THE CROSS SECTION, READ ROW WISE

0.0	C.0	0.0	0.30000E 02	0.30000E 02	0.30000E 02
0.60000E 02	C.60000E 02	0.60000E 02	0.90000E 02	0.90000E 02	0.90000E 02
0.12000E 03	C.12000E 03	0.12000E 03	0.15000E 03	0.15000E 03	0.15000E 03
0.18000E 03	C.18000E 03	0.18000E 03	0.21000E 03	0.21000E 03	0.21000E 03
0.24000E 03	C.24000E 03	0.24000E 03	0.27000E 03	0.27000E 03	0.27000E 03
0.30000E 03	C.30000E 03	0.30000E 03	0.33000E 03	0.33000E 03	0.33000E 03

ANGULAR ORIENTATION OF THE GAGES, READ ROW WISE

0.0	C.60000E 02	0.12000E 03	0.0	0.60000E 02	0.12000E 03
0.60000E 02	C.60000E 02	0.12000E 03	0.0	0.60000E 02	0.12000E 03
0.60000E 02	C.60000E 02	0.12000E 03	0.0	0.60000E 02	0.12000E 03
0.60000E 02	C.60000E 02	0.12000E 03	0.0	0.60000E 02	0.12000E 03
0.60000E 02	C.60000E 02	0.12000E 03	0.0	0.60000E 02	0.12000E 03

READINGS OF THE GAGE ELEMENTS, READ ROW WISE.

0. 15134E-04	C. 63164E-06	0. 19596E-06	0. 14907E-04	0. 53904E-06	0. 33688E-06
0. 14833E-04	C. 40585E-06	0. 50206E-06	0. 15126E-04	0. 29224E-06	0. 63844E-06
0. 15008E-04	C. 22246E-06	0. 73175E-06	0. 14883E-04	0. 22505E-06	0. 73152E-06
0. 15183E-04	C. 29368E-06	0. 64296E-06	0. 14880E-04	0. 40417E-06	0. 49667E-06
0. 14843E-04	C. 53846E-06	0. 33419E-06	0. 15091E-04	0. 64980E-06	0. 19406E-06
0. 15088E-04	C. 71787E-06	0. 11202E-06	0. 15327E-04	0. 71808E-06	0. 11338E-06

EXPANSION WITH 11 FOURIER COEFFICIENTS

$KS = 0$ KS IS AN INDEX INDICATING TYPE OF SOLUTION: 0 FOR NORMAL, 1 FOR SINGULAR

FOURIER COEFFICIENTS

CONSTANTS

TANG.	0.45167E C3
LONG.	0.30279E C1
SHEAR	0.67538E CC

	COS 1(THETA)	SIN 1(THETA)	COS 2(THETA)	SIN 2(THETA)	COS 3(THETA)	SIN 3(THETA)
TANG.	0.16451E 01	-0.94166E 00	0.10407E 01	-0.36259E 01	-0.60751E 00	-0.22247E 01
LONG.	-C.11111E C1	0.10538E 01	-0.76545E-01	0.86755E-01	-0.51607E-01	0.13186E 00
SHEAR	C.53511E C1	-C.54097E 01	-0.47720E-01	0.44068E-01	-0.65953E-C1	-0.32323E-01

	COS 4(THETA)	SIN 4(THETA)	COS 5(THETA)	SIN 5(THETA)
TANG.	0.31820E 01	-0.17485E-01	-0.21132E 01	-0.46379E 00
LONG.	-C.214C2E CC	-0.24257E-01	-0.92955E-02	0.12406E-01
SHEAR	-0.22891E-01	-0.177C9E-03	-0.55914E-01	0.34983E-01

THETA = 240 DEGREES	POINT NO.	241	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E C3	0.24770E 01	0.25407E 01
COMPUTED	0.44647E C3	0.28761E 01	0.27506E 01
THETA = 250 DEGREES	POINT NO.	251	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E C3	0.22133E 01	0.37883E 01
COMPUTED	0.44979E C3	0.25514E 01	0.39775E 01
THETA = 260 DEGREES	POINT NO.	261	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E C3	0.19702E 01	0.49382E 01
COMPUTED	0.45222E C3	0.22239E 01	0.50755E 01
THETA = 270 DEGREES	POINT NO.	271	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E C3	0.17552E 01	0.59553E 01
COMPUTED	0.45299E C3	0.19561E 01	0.60426E 01
THETA = 280 DEGREES	POINT NO.	281	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E C3	0.15748E 01	0.68088E 01
COMPUTED	0.45262E C3	0.17848E 01	0.68838E 01
THETA = 290 DEGREES	POINT NO.	291	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E C3	0.14345E 01	0.74726E 01
COMPUTED	0.45243E C3	0.17031E 01	0.75850E 01
THETA = 300 DEGREES	POINT NO.	301	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E C3	0.13385E 01	0.79267E 01
COMPUTED	0.45347E C3	0.16666E 01	0.81011E 01
THETA = 310 DEGREES	POINT NO.	311	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E C3	0.12898E 01	0.81573E 01
COMPUTED	0.45576E C3	0.16207E 01	0.83706E 01
THETA = 320 DEGREES	POINT NO.	321	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E C3	0.12897E 01	0.81573E 01
COMPUTED	0.45836E C3	0.15346E 01	0.83472E 01
THETA = 330 DEGREES	POINT NO.	331	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E C3	0.13385E 01	0.79267E 01
COMPUTED	0.45993E C3	0.14234E 01	0.80272E 01
THETA = 340 DEGREES	POINT NO.	341	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E C3	0.14345E 01	0.74726E 01
COMPUTED	0.45968E C3	0.13451E 01	0.74556E 01
THETA = 350 DEGREES	POINT NO.	351	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E C3	0.15748E 01	0.68088E 01
COMPUTED	0.45770E C3	0.13746E 01	0.67035E 01

EXPANSION WITH 5 FOURIER COEFFICIENTS

K_S = 0 K_S IS AN INDEX INDICATING TYPE OF SOLUTION: 0 FOR NORMAL, 1 FOR SINGULAR

FOURIER COEFFICIENTS

CONSTANTS

TANG.	0.45167E 03
LONG.	2.30279E 01
SHEAR	0.67538E 00

COS 1(THETA)	SIN 1(THETA)	COS 2(THETA)	SIN 2(THETA)
0.16451E 01	-0.94167E 00	0.10407E 01	-0.36258E 01
-0.11111E 01	0.10538E 01	-0.76546E-01	0.86754E-01
2.53511E 01	-2.54097E 01	-0.47720E-01	0.44968E-01

THETA = 240 DEGREES	POINT NO.	241	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.24770E 01	0.25407E 01
COMPUTED	0.44800E 03	0.27843E 01	0.27467E 01
THETA = 250 DEGREES	POINT NO.	251	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.22133E 01	0.37883E 01
COMPUTED	0.44886E 03	0.25321E 01	0.39935E 01
THETA = 260 DEGREES	POINT NO.	261	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.19702E 01	0.49382E 01
COMPUTED	0.45009E 03	0.22847E 01	0.51336E 01
THETA = 270 DEGREES	POINT NO.	271	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.17552E 01	0.59553E 01
COMPUTED	0.45157E 03	0.20507E 01	0.61327E 01
THETA = 280 DEGREES	POINT NO.	281	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.15748E 01	0.68088E 01
COMPUTED	0.45314E 03	0.18395E 01	0.69618E 01
THETA = 290 DEGREES	POINT NO.	291	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.14345E 01	0.74726E 01
COMPUTED	0.45465E 03	0.16605E 01	0.75972E 01
THETA = 300 DEGREES	POINT NO.	301	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.13385E 01	0.79267E 01
COMPUTED	0.45592E 03	0.15229E 01	0.80215E 01
THETA = 310 DEGREES	POINT NO.	311	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.12898E 01	0.81573E 01
COMPUTED	0.45684E 03	0.14343E 01	0.82239E 01
THETA = 320 DEGREES	POINT NO.	321	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.12897E 01	0.81573E 01
COMPUTED	0.45728E 03	0.14007E 01	0.82001E 01
THETA = 330 DEGREES	POINT NO.	331	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.13385E 01	0.79267E 01
COMPUTED	0.45722E 03	0.14253E 01	0.79524E 01
THETA = 340 DEGREES	POINT NO.	341	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.14345E 01	0.74726E 01
COMPUTED	0.45666E 03	0.15090E 01	0.74891E 01
THETA = 350 DEGREES	POINT NO.	351	
STRESSES	TANG.	LONG.	SHEAR
ACTUAL	0.45455E 03	0.15748E 01	0.68088E 01
COMPUTED	0.45567E 03	0.16491E 01	0.68246E 01

EXPANSION WITH 3 FOURIER COEFFICIENTS

KS = 0 KS IS AN INDEX INDICATING TYPE OF SOLUTION: 0 FOR NORMAL, 1 FOR SINGULAR

FOURIER COEFFICIENTS

CONSTANTS
0.45167E 03
0.30279E 01
0.67538E 00

COS 1(THETA) SIN 1(THETA)
0.16451E 01 -0.94167E 00
0.11111E 01 0.10533E 01
0.53511E 01 -0.54097E 01

THETA = 240 DEGREES	STRESSES	TANG.	POINT NO. 241	
	ACTUAL	0.45455E 03	0.24770E 01	SHEAR
	COMPUTED	0.45166E 03	0.26709E 01	0.25407E 01 0.26847E 01
THETA = 250 DEGREES	STRESSES	TANG.	POINT NO. 251	
	ACTUAL	0.45455E 03	0.22133E 01	SHEAR
	COMPUTED	0.45199E 03	0.24177E 01	0.37883E 01 0.39286E 01
THETA = 260 DEGREES	STRESSES	TANG.	POINT NO. 261	
	ACTUAL	0.45455E 03	0.19702E 01	SHEAR
	COMPUTED	0.45231E 03	0.21831E 01	0.49382E 01 0.50736E 01
THETA = 270 DEGREES	STRESSES	TANG.	POINT NO. 271	
	ACTUAL	0.45455E 03	0.17552E 01	SHEAR
	COMPUTED	0.45261E 03	0.19741E 01	0.59553E 01 0.60850E 01
THETA = 280 DEGREES	STRESSES	TANG.	POINT NO. 281	
	ACTUAL	0.45455E 03	0.15748E 01	SHEAR
	COMPUTED	0.45288E 03	0.17972E 01	0.68088E 01 0.69321E 01
THETA = 290 DEGREES	STRESSES	TANG.	POINT NO. 291	
	ACTUAL	0.45455E 03	0.14345E 01	SHEAR
	COMPUTED	0.45311E 03	0.16577E 01	0.74726E 01 0.75890E 01
THETA = 300 DEGREES	STRESSES	TANG.	POINT NO. 301	
	ACTUAL	0.45455E 03	0.13385E 01	SHEAR
	COMPUTED	0.45330E 03	0.15597E 01	0.79267E 01 0.80358E 01
THETA = 310 DEGREES	STRESSES	TANG.	POINT NO. 311	
	ACTUAL	0.45455E 03	0.12898E 01	SHEAR
	COMPUTED	0.45345E 03	0.15064E 01	0.81573E 01 0.82590E 01
THETA = 320 DEGREES	STRESSES	TANG.	POINT NO. 321	
	ACTUAL	0.45455E 03	0.12897E 01	SHEAR
	COMPUTED	0.45353E 03	0.14994E 01	0.81573E 01 0.82518E 01
THETA = 330 DEGREES	STRESSES	TANG.	POINT NO. 331	
	ACTUAL	0.45455E 03	0.13385E 01	SHEAR
	COMPUTED	0.45356E 03	0.15388E 01	0.79267E 01 0.80144E 01
THETA = 340 DEGREES	STRESSES	TANG.	POINT NO. 341	
	ACTUAL	0.45455E 03	0.14345E 01	SHEAR
	COMPUTED	0.45353E 03	0.16234E 01	0.74726E 01 0.75540E 01
THETA = 350 DEGREES	STRESSES	TANG.	POINT NO. 351	
	ACTUAL	0.45455E 03	0.15748E 01	SHEAR
	COMPUTED	0.45345E 03	0.17507E 01	0.68088E 01 0.68846E 01

MODULUS OF ELASTICITY C.30000E 08PSI
 SHEAR MODULUS OF ELASTICITY 0.11538F 08 PSI
 POISSON RATIO 0.3330E 0C

NUMBER OF GAGES 36
 TRANSVERSE SENSITIVITY FACTOR OF THE GAGES = 1.8
 NUMBER OF GAGES FUNCTIONING PROPERLY 36
 GAGES FUNCTIONING IMPROPERLY ••• NONE

ANGULAR POSITION OF THE GAGE IN THE CROSS SECTION, READ ROW WISE

0.0	C. C	0.0	0.30000E 02	0.30000E 02	0.30000E 02
0.60000E 02	C. 60000E C2	0.60000E 02	0.90000E 02	0.90000E 02	0.90000E 02
0.12000E 03	C. 12000E 03	0.12000E 03	0.15000E 03	0.15000E 03	0.15000E 03
0.18000E 03	C. 18000E 03	0.18000E 03	0.21000E 03	0.21000E 03	0.21000E 03
0.24000E 03	C. 24000E 03	0.24000E 03	0.27000E 03	0.27000E 03	0.27000E 03
0.30000E 03	C. 30000E 03	0.30000E 03	0.33000E 03	0.33000E 03	0.33000E 03

ANGULAR ORIENTATION OF THE GAGES, READ ROW WISE

0.0	C. 60000E 02	0.12000E 03	0.0	0.60000E 02	0.12000E 03
0.0	C. 60000E 02	0.12000E 03	0.0	0.60000E 02	0.12000E 03
0.0	C. 60000E 02	0.12000E 03	0.0	0.60000E 02	0.12000E 03
0.0	C. 60000E 02	0.12000E 03	0.0	0.60000E 02	0.12000E 03
0.0	C. 60000E 02	0.12000E 03	0.0	0.60000E 02	0.12000E 03

READINGS OF THE GAGE ELEMENTS, READ ROW WISE. ERROR ADDED %

0.15091E-04	C. 74191E-06	0.29935E-06	0.15084E-04	0.63129E-06	0.44247E-06
0.15076E-04	C. 50355E-06	0.60773E-06	0.15069E-04	0.39292E-06	0.75086E-06
0.15065E-04	C. 32905E-06	0.83349E-06	0.15065E-04	0.32905E-06	0.83349E-06
0.15069E-04	C. 39292E-06	0.75086E-06	0.15076E-04	0.50355E-06	0.60773E-06
0.15084E-04	C. 63129E-06	0.44247E-06	0.15091E-04	0.74191E-06	0.29935E-06
0.15095E-04	C. 80579E-06	0.21672E-06	0.15095E-04	0.80579E-06	0.21672E-06

COEFFICIENTS A, READ ROW WISE

0.33237E-07	C.10585E-C8	0.10585E-08	0.33237E-07	0.10585E-08	0.10585F-08
0.33237E-07	C.10585E-08	0.10585E-08	0.33237E-07	0.10585E-08	0.10585E-08
0.33237E-07	C.10585E-08	0.10585E-08	0.33237E-07	0.10585E-08	0.10585E-08
0.33237E-07	C.10585E-08	0.10585E-08	0.33237E-07	0.10585E-08	0.10585E-08
0.33237E-07	C.10585E-08	0.10585E-08	0.33237E-07	0.10585E-08	0.10585E-08
0.33237E-07	C.10585E-08	0.10585E-08	0.33237E-07	0.10585E-08	0.10585F-08

COEFFICIENTS B, READ ROW WISE

-0.96676E-08	C.22511E-07	0.22511E-07	-0.96676E-08	0.22511E-07	0.22511E-07
-0.96676E-08	C.22511E-07	0.22511E-07	-0.96676E-08	0.22511E-07	0.22511E-07
-0.96676E-08	C.22511E-07	0.22511E-07	-0.96676E-08	0.22511E-07	0.22511E-07
-0.96676E-08	C.22511E-07	0.22511E-07	-0.96676E-08	0.22511E-07	0.22511E-07
-0.96676E-08	C.22511E-07	0.22511E-07	-0.96676E-08	0.22511E-07	0.22511E-07
-0.96676E-08	C.22511E-07	0.22511E-07	-0.96676E-08	0.22511E-07	0.22511E-07

COEFFICIENTS C, READ ROW WISE

-0.0	C.37156E-07	-0.37156E-07	-0.0	0.37156E-07	-0.37156E-07
-0.0	C.37156E-07	-0.37156E-07	-0.0	0.37156E-07	-0.37156E-07
-0.0	C.37156E-07	-0.37156E-07	-0.0	0.37156E-07	-0.37156E-07
-0.0	C.37156E-07	-0.37156E-07	-0.0	0.37156E-07	-0.37156E-07
-0.0	C.37156E-07	-0.37156E-07	-0.0	0.37156E-07	-0.37156E-07
-0.0	C.37156E-07	-0.37156E-07	-0.0	0.37156E-07	-0.37156E-07

EXPANSION WITH 11 FOURIER COEFFICIENTS

KS = 0 KS IS AN INDEX INDICATING TYPE OF SOLUTION: 0 FOR NORMAL, 1 FOR SINGULAR

FOURIER COEFFICIENTS

	CONSTANTS	SIN 1(THETA)	COS 2(THETA)	SIN 2(THETA)	COS 3(THETA)	SIN 3(THETA)
TANG.	0.45455E C3	0.32390E-04	0.56746E-04	-0.58088E-04	0.54121E-04	0.11337E-04
LONG.	0.28944E C1	0.11385E 01	0.43307E-04	-0.76463E-04	0.74386E-05	0.55022E-04
SHEAR	C.56525E CC	-0.53861E C1	-0.53861E 01	0.52491E-05	0.28910E-05	-0.14306E-05
						-0.13928E-05
	COS 4(THETA)	SIN 4(THETA)	COS 5(THETA)	SIN 5(THETA)		
TANG.	0.76199E-04	0.11885E-04	0.38986E-04	-0.78963E-05		
LONG.	C.41156E-C4	-0.55075E-04	-0.13349E-04	0.45193E-04		
SHEAR	0.10598E-05	-0.10318E-05	C.22680E-06	-0.69870E-06		

THETA =	0 DEGREES	POINT NO.	1	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E C3	0.17552E 01	0.59553E 01	
COMPUTED	0.45455E C3	0.17558E 01	0.59553E 01	
THETA =	10 DEGREES	POINT NO.	11	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E C3	0.19702E 01	0.49382E 01	
COMPUTED	0.45455E C3	0.19707E 01	0.49382E 01	
THETA =	20 DEGREES	POINT NO.	21	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E C3	0.22133E 01	0.37884E 01	
COMPUTED	0.45455E C3	0.22137E 01	0.37884E 01	
THETA =	30 DEGREES	POINT NO.	31	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E C3	0.24770E 01	0.25407E 01	
COMPUTED	0.45455E C3	0.24774E 01	0.25407E 01	
THETA =	40 DEGREES	POINT NO.	41	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E C3	0.27534E 01	0.12331E 01	
COMPUTED	0.45455E C3	0.27538E 01	0.12331E 01	
THETA =	50 DEGREES	POINT NO.	51	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E C3	0.30341E 01	-0.94609E-01	
COMPUTED	0.45455E C3	0.30345E 01	-0.94604E-01	
THETA =	60 DEGREES	POINT NO.	61	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E C3	0.33104E 01	-0.14022E 01	
COMPUTED	0.45455E C3	0.33110E 01	-0.14022E 01	
THETA =	70 DEGREES	POINT NO.	71	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E C3	0.35742E 01	-0.26498E 01	
COMPUTED	0.45455E C3	0.35748E 01	-0.26498E 01	
THETA =	80 DEGREES	POINT NO.	81	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E C3	0.38172E 01	-0.37997E 01	
COMPUTED	0.45455E C3	0.38180E 01	-0.37997E 01	
THETA =	90 DEGREES	POINT NO.	91	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E C3	0.40322E 01	-0.48168E 01	
COMPUTED	0.45455E C3	0.40331E 01	-0.48168E 01	
THETA =	100 DEGREES	POINT NO.	101	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E C3	0.42126E 01	-0.56702E 01	
COMPUTED	0.45455E C3	0.42135E 01	-0.56702E 01	
THETA =	110 DEGREES	POINT NO.	111	
STRESSES	TANG.	LONG.		SHEAR
ACTUAL	0.45455E C3	0.43530E 01	-0.63341E 01	
COMPUTED	0.45455E C3	0.43537E 01	-0.63341E 01	

APPENDIX C

DIGITAL COMPUTER PROGRAM FOR SECTION 3 AND NUMERICAL RESULTS

Subroutine MACUTO presented in this Appendix provides a method for applying the theory developed in Section 3. Data for simulated situations can be obtained from subroutine CANTIL presented in Appendix A.

MACUTO has been programmed following closely the development of the method. Self explanatory comment statements are included, which detail the type of data to be used and the procedural sequence of the method. Print-outs present the data processed and the computed loading.

Use of the subroutines is made by a brief main program called TIBISAY. This program merely dimensions the subroutines, accepts and prepares the input data, and turns control over to subroutine MACUTO. The card arrangement of the source deck is similar to that shown in Fig. B.1.

Some sample results of the application made of the programs as specified in Section 3 are presented following the card listings of the programs.

//TIBISAY JIN '1224' • TANIDA, JESUS A. • MSCUL.VFL = 1

//FORTRAN EXEC FORTCLG

OCOCO10

DIMENSION AF1(3), XM1(3), AF(3), XM(3), F(3), X(3), FWHT(3),
1 X(2), X(2), FR(3), EZ(3), FT(3), ER(3), ALL1(3), 360, 5,
2 SHZT(3), SEZ(3), SATZ(3), SHFZT(3), SPR2(3), SPT(3),
DIMENSION F(3,3), FRC(3), XMOM(3), THETA(3), ST(3), SL(3), SH(3), Y(3,3)
DIMENSION A1(3), A2(3), A3(3), A4(3), F3(3), F4(3), F5(3), F6(3),
1 XM3(3), XM4(3), XM5(3), XM6(3), B1(3), B2(3), B3(3), B4(3), B5(3), B6(3)

C SPECIFY POISSON'S RATIO V, EXTERNAL AND INTERNAL PIPE DIAMETERS DO
C AND DISSYULAR POSITION THETA OF THE POINTS AND STRESS COMPONENTS:
C TANGENTIAL ST., LONGITUDINAL SL AND SHARP SH OF THE THREE POINTS 1,
C 2, AND 3.

C IF THE MATHEMATICAL MODEL OF A PIPE IS USED, OUTPUT ARGUMENTS OF
C SUBROUTINE CANTIL, XM1 AND ALL1 CAN BE USED FOR THIS.

C CALL CANTIL(F1), RI, XE, G, XNU, SPECW, D, XL1, NPOINT, APFA, ALL1)

V=XNU

DC=RO+RC

DI=RI+RI

THETA(1)=0

THETA(2)=120

THETA(3)=240

ST(1)=ALL1(1,1,1)

ST(2)=ALL1(1,121,1)

ST(3)=ALL1(1,241,1)

SL(1)=ALL1(2,1,1)

SL(2)=ALL1(2,121,1)

SL(3)=ALL1(2,241,1)

SH(1)=ALL1(3,1,1)

SH(2)=ALL1(3,121,1)

SH(3)=ALL1(3,241,1)

C PREPARE STRESSES FOR INPUT TO MACUTO.

DO 1 I = 1, 2
1 Y(1,1) = S1(I)
1 Y(2,1) = SL(I)
1 Y(3,1) = SH(I)

EVALUATE THE LOADING AT THE CROSS SECTION OF PIPE.

CALL MACUTC(DC,CI,V,Y,THETA,D,PIPEF,XMON)

C
C
C
C
C

END

SUMMING MACUT001, DI, V, ALL, THETA, D, F, YM)

TO FIND THE LOADING ACTING THROUGH THE CROSS SECTION OF A PIPE FROM THE STRESS COMPONENTS AT THREE POINTS OF THE EXTERNAL CIRCUMFERENCE OF THE CROSS SECTION.

D0 AND DI ARE EXTERNAL AND INTERNAL DIAMETERS OF THE PIPE RESPECTIVELY.
V IS PRESSURE RATIO.
ALL IS A 3X3 ARRAY CONTAINING THE STRESS COMPONENTS FOR THE THREE POINTS.
THETA IS AN ARRAY CONTAINING THE ANGULAR POSITIONS OF THE POINTS.
P IS THE INFERRED INTERNAL FORCE.
F IS THE INFERRED FORCE.
M IS THE INFERRED MOMENT.

DIMENSION F(3), XM(3), ALL(3, 3), THETA(3)
ABC(F(A1,A2,A3,B1,B2,B3) = A1*B1+A2*B2+A3*B3
WRITE(6, 50)
FORMAT(1H1, 7(/), 15X, 'INFERRED LOADING' //)
500
R0 = D0/2.0
R02 = D0*D0/4.0
RI2 = D1*D1/4.0
PI = 3.14159265
ARFA = PI*(R02 - RI2)
ERTIA = ARFA*(R02 + RI2)/4.0
THETA2 = PI/180.0
ENE = V + 1.5
D = ((2.0*ENE - 2.0)*R02 + 2.0*ENE*RI2)/(4.0*ERTIA*(1.0 + V))
C IDENTIFY THE STRESS COMPONENTS, TANGENTIAL SPTT, LONGITUDINAL S
AND SHEAR T FOR POINTS 1, 2, AND 3 FROM ARRAY ALL.

SPTT1 = ALL(1, 1)
S1 = ALL(2, 1)
T1 = ALL(3, 1)
SPTT2 = ALL(1, 2)
S2 = ALL(2, 2)
T2 = ALL(3, 2)
SPTT3 = ALL(1, 3)
S3 = ALL(2, 3)
T3 = ALL(3, 3)
MACUT001
MACUT002
MACUT003
MACUT004
MACUT005
MACUT006
MACUT007
MACUT008
MACUT009
MACUT010
MACUT011
MACUT012
MACUT013
MACUT014
MACUT015
MACUT016
MACUT017
MACUT018
MACUT019
MACUT020
MACUT021
MACUT022
MACUT023
MACUT024
MACUT025
MACUT026
MACUT027
MACUT028
MACUT029
MACUT030
MACUT031
MACUT032
MACUT033
MACUT034
MACUT035
MACUT036
MACUT037
MACUT038
MACUT039
MACUT040
MACUT041
MACUT042
MACUT043
MACUT044
MACUT045
MACUT046
MACUT047
MACUT048

CONVERT ANGULAR DEGREES TO RADIANS AND FIND DIFFERENCES BETWEEN ANGLES. FIND DIFFERENCES BETWEEN STRESS COMPONENTS.

```

C      P = ( SPTT1 + SPTT2 + SPTT3 )*( R()2/R12 - 1.2 )/5.0
C
C      PREPARE PRINT-CUTS OF THE INFORMATION PROCESSFD AND THE INFERFD
C      LOADING.
C
C      WRITE(6, 2)  THETA(1), (ALL(I,1), I = 1, 3),
C      1          THETA(2), (ALL(I,2), I = 1, 3),
C      2          THETA(3), (ALL(I,3), I = 1, 3)
C      1          *SHEAR, 15X, *STRESSES, 28X, *TANG, *LONG, *DEGREES, 3E2.5 /, 17X,
C      1          WRITE(6, 3)  P, XM
C      2          FORMAT( //, 15X, *INFERRED PRESSURE, E20.5, PSI /,
C      1          FORMAT( //, 15X, *INFERRED FORCE, 3E20.5, LBS /,
C      2          15X, *INFERRED MOMENT, 3E20.5, LR-IN /,
C
C      RETURN
END

```

INTERIOR LADING

STATE OF STRESS	TANG.	LFRG.	SHEAR
INITIAL = 24.0 0 ^o DEGREES	0.45455F 03	-0.97505F 02	0.59553F 01
INT'LAD = 120.0 0 ^o DEGREES	0.45455F 03	0.16057F 02	-0.20473E 02
INT'LAD = 24.0 0 ^o DEGREES	0.45455F 03	0.74187E 02	0.16226F 02

INFERRED PRESSURE INFLATED FOLIAGE INFLATED MOUNT	6.1C200E 03 PSI 6.3C33CE 03 -6.2A017F 04	2.19000E 03 2.75009E 04	0.99999E 02 0.10000E 03 LB-IN
---	--	----------------------------	-------------------------------------

RADIAL DISTANCE OF POINTS = 6.0000 INCHES			
LOADS APPLIED TO THE TIP			
INTERNAL PRESSURE = 100.00 PSI	100.00	100.00	100.00 LB
APPLIED FORCE = 100.00	100.00	100.00	100.00 LB-IN
APPLIED MOMENT = 100.00	100.00	100.00	100.00 LB-IN
DISTANCE FROM THE TIP TO THE CROSS SECTION = 30.00 INCHES			
CAPS ACTING ON THE CROSS SECTION			
INTERNAL PRESSURE = 100.00 PSI	100.00	100.00	100.00 LB
APPLIED FORCE = 393.34	-393.34	750.00	750.00 LB-IN
APPLIED MOMENT = -200.00	200.00	750.00	750.00 LB-IN

The stresses used as input for this inference were taken from this data which is similar to that shown in pg. • The inferred loading is in agreement with the actual one acting on the cross section.

APPENDIX D

Subroutine DSIMQ of the Computer Facility Center of the Naval Postgraduate School, to solve linear simultaneous equations.

DSIM002

PURPOSE
OBTAIN SOLUTION OF A SET OF SIMULTANEOUS LINEAR EQUATIONS,
 $A \times B$

USAGE CALL NUMBER MARKS

DESCRIPTION OF PARAMETERS

A AND B MUST BE REAL*.
A - MATRIX OF COEFFICIENTS STORED COLUMNWISE. THESE ARE DESTROYED IN THE COMPUTATION. THE SIZE OF MATRIX A IS N BY N.
B - VECTOR OF ORIGINAL CONSTANTS (LENGTH N). THESE ARE REPLACED BY FINAL SOLUTION VALUES. VECTOR X.
N - NUMBER OF EQUATIONS AND VARIABLES.

UNIFORM BICORNAL SOLUTION OF SINGULAR SET OF EQUATIONS

REMARKS

MATRIX MUST BE SINGULAR. SOLUTION VALUES ARE MEANINGLESS.
IF MATRIX IS SINGULAR, AN ALTERNATIVE SOLUTION MAY BE OBTAINED BY USING MATRIX INVERSION (MINV) AND MATRIX PRODUCT (MPROD).

SUBROUTINES AND FUNCTIONS AND FUNCTION SUBPROGRAMS REQUIRED ... NONE

METHOD OF SOLUTION IS BY ELIMINATION USING LARGEST PIVOTAL DIVISOR. EACH STAGE OF ELIMINATION CONSISTS OF INTERCHANGING ROWS WHEN NECESSARY TO AVOID DIVISION BY ZERO OR SMALL ELEMENTS.

THE FORWARD SOLUTION TO OBTAIN VARIABLE N IS DONE IN N STAGES. THE BACK SOLUTION FOR THE OTHER VARIABLES IS CALCULATED BY SUCCESSIVE SUBSTITUTIONS. FINAL SOLUTION VALUES ARE DEVELOPED IN VECTOR B WITH VARIABLE 1 IN B(1), VARIABLE 2 IN B(2), VARIABLE N IN B(N).

IF NO PIVOT CAN BE FOUND EXCEEDING A TOLERANCE OF 0.C, THE MATRIX IS CONSIDERED SINGULAR AND KS IS SET TO 1. THIS TOLERANCE CAN BE MODIFIED BY REPLACING THE FIRST STATEMENT.

```

C SUBROUTINE DSIMQ(A,B,N,KS)
C RFAL*B,A,BIGA,TUL,SAVF,DAVS
C DIMENSION A(1),B(1)

FORWARD SITUATION

TOL=0.000
TOL=0.0
KS=0
JJ=-N
DO 65 J=1,N
JY=J+1
JJ=JJ+N+1
BIGA=0.0
BIGA=0.0
IT=JJ-J
DO 30 I=J,N

C SEARCH FOR MAXIMUM COEFFICIENT IN COLUMN

IJ=IT+1
IF (DABS(BIGA)-DABS(A(IJ))) 20,30,30
IF (ABS(BIGA)-ABS(A(IJ))) 20,30,30
20 BIGA=A(IJ)
IMAX=I
CONTINUE

C TEST FOR PIVOT LESS THAN TOLERANCE (SINGULAR MATRIX)

IF (DABS(BIGA)-TOL) 35,35,40
IF (ABS(BIGA)-TOL) 35,35,40
35 KS=1
RETURN

C INTERCHANGE ROWS IF NECESSARY

40 I1=J+N*(J-2)
I1=IMAX-J
DO 50 K=J,N
I1=I1+N
I2=I1+IT
SAVE=A(I1)
A(I1)=A(I2)

```

```

C C C DIVIDE EQUATION BY LEADING COEFFICIENT
C C C
      50 A(I,I)=A(I,I)/B(I,G)
      SAVE=B(I,MAX)
      B(I,MAX)=B(J)
      B(J)=SAVE/3*IGA

C C ELIMINATE NEXT VARIABLE
C C
      55 IF(J=N) 55,70,55
      IQS=N*(J-1)
      DO 65 IX=JY,N
      IXJ=IQS+IX
      IT=J-IX
      DO 60 JX=JY,N
      IXJX=N*(JX-1)+IX
      JXJ=IXJX+IT
      A(IXJX)=A(IXJX)-(A(IXJ)*A(JJX))
      65 B(IX)=B(IX)-(B(J)*A(IXJ))

C C BACK SOLUTION
C C
      70 NY=N-1
      IT=N*N
      DO 80 J=1,NY
      IA=IT-J
      IB=N-J
      IC=N
      DO 80 K=1,J
      B(IB)=B(IB)-A(IA)*B(IC)
      IA=IA-N
      IC=IC-1
      80 RETURN
      END

```

APPENDIX E

Effects of Changing the Number of Fourier Coefficients and the Amount and Quality of the Data in the Application of the Theory Developed in Section 2.

The theory developed in Section 2 of this thesis estimates Fourier coefficients of the distributions of axial, tangential, and shear stresses assuming that enough data has been obtained by placing a sufficient number of strain gage elements around the external surface of a cross section of pipe. The correctness of the estimated coefficients depends on their number and on the amount and quality of the data. Depending on the relative orientation of the gage elements, they can pick information useful for determining the coefficients of all the distributions, or, if they are arranged in certain disadvantageous ways, they may be able to provide information for none, or only some of the coefficients.

The requirement of inferring a given number N of Fourier coefficients, demands having, at a minimum, a corresponding number $M = M(N)$ of valid data items. If more data are available, there is a redundancy of data which permits more accurate determination of the N coefficients.

Specific rules for the necessary number and disposition of gage elements to obtain a specified number of correct coefficients are not available, but they probably could be worked out. Some idea of the gage elements requirements can be obtained by testing actual cases and comparing results with correct ones obtained from other methods. The digital computer programs included in this thesis can be used as they are to perform such tests. The procedure to follow is to simply specify in subroutine ZULIA the number of gage elements, their angular positions and angular orientations around the cross section, and the number of Fourier coefficients to be computed. The print-out of this program presents the

distributions obtained in this way, shown adjacent to the theoretically correct ones, so that comparisons can be readily done.

A preliminary survey of gage elements arrangements was made using that procedure, applied to data with no added random errors. Results obtained are presented following these pages. Many of these results might have been anticipated from theoretical considerations. The following nomenclature is used when applicable:

(a) No. of coefficients

3 to 11: the results presented were obtained using from three to eleven Fourier coefficients per series. These results apply to that number of coefficients.

7: results apply only to seven coefficients.

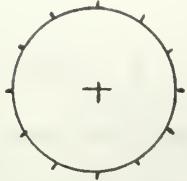
(b) Results

good: indicates that the coefficients obtained describe correctly the tangential, axial and shear stress distributions.

bad: The computed coefficients do not describe either of the distributions.

only shear good: good coefficients were obtained only for the shear stress distribution.

Inasmuch as the data were without added errors, the recovery should be and actually was excellent or no good at all; accordingly, there is nothing subjective in the judgment "good" or "bad".



Gage elements located at 12 points
equally spaced 30° around the external
circumference of the cross section.

gage elements
arrangement
at each point

No. of Fourier
coefficients per series

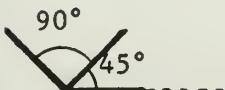
results



T axis

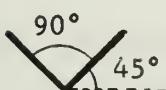
3 to 11

good



3 to 11

good



3 to 11

only shear good



3 to 11

bad



3 to 11

bad



3 to 11

good



3 to 11

only shear good



3 to 11

bad

II)

Gage elements located at 8 points
equally spaced 45° around the external
circumference of the cross section.

gage elements
arrangement
at each point

No. of Fourier
coefficients per series

results



3 to 7

good

same

9 and 11

bad



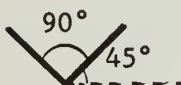
3 to 7

good

same

9 and 11

bad



3 to 7

only shear good

same

9 and 11

bad



3 to 11

bad



3 to 11

bad

III)

One gage element every 10° around the external circumference of the cross section, totaling 36 elements. The first element at 0° from the tangential direction, and every consecutive one, incrementing this angle by 10° .

from 3 to 11 coefficients: bad results

IV)

One gage element every 10° around the external circumference of the cross section, totaling 36 elements. The first element at 0° from the tangential direction, and every consecutive one, incrementing this angle by 50° .

from 3 to 9 coefficients: good results

11 coefficients: bad results

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13. ABSTRACT

This thesis presents a method for estimating the distribution of stress components around the outer circumference of a cross section of a pipe, from the readings of strain gage elements arbitrarily positioned and oriented around this circumference. A least-squares procedure is used to obtain best estimates of the coefficients of Fourier expansions describing such distributions. A digital computer program was developed for applying the method. Data for testing the method and program were generated by a computer program using the best available theory of stress analysis in pipes. Methods for adding random errors to the data were adapted and used for closer simulation of actual situations.

A second problem treated in this thesis is that of inferring the loading acting through the cross section of a straight pipe of concentric bore from known stresses at points on the external surface of the cross section which is presumed to be distant from stress concentrations. It is shown that this inference can be made from stresses at only three points of the cross section. A digital computer program was developed to do this.

14.

KEY WORDS

	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Strain Analysis						
Pipe Stresses						
Strain Gage Data						
Data Reduction						
Experiments on Piping						
Optimal Data Analysis						

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